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PROJECT NS 715-102, DAVID TAYLOR MODEL BASIN

Hydrodynamics Laboratory
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Pasadena, California

Report No. 47-4
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ABSTRACT

This paper investigates in a rather idealized way the different properties of fully cavitating and fully wetted hydrofoils in order to clarify the relative hydrodynamic merits of each insofar as this is possible in the present state of the art. The discussion is mainly based on a recent theory, together with some experimental data, on the hydrodynamics of two-dimensional fully cavitating hydrofoils. A number of quantitative comparisons between the fully cavitating and fully wetted two-dimensional foils have been made to bring out the different effects of such design parameters as attack angle, camber, submergence and speed on the hydrofoil in the two regimes. In addition, some of the effects which modify the two-dimensional comparison are surveyed and roughly estimated wherever possible. The consequences of air ventilation (which is closely related to fully cavitating flow) are discussed, especially as applied to the supporting struts, from the standpoint of whether or not it should be avoided. Finally, after a few remarks on some practical aspects of the problem, a rough comparison is made from the economy point of view to indicate by an example how a criterion might be chosen for one or the other type of operation. From this and the preceding calculations it is conjectured that there is strong reason to believe that the fully cavitating type of operation will be advantageous in some circumstances, but it is emphasized that more experience must be accumulated for operation in both regimes before any practical criteria can be specified.

I. INTRODUCTION

The application of hydrofoils to surface water craft has been the subject of renewed interest in recent years because such foil-equipped boats may have a potential superiority over conventional types in the higher speed ranges.[1,2] A hydrofoil boat might consist, for example, of a somewhat conventional hull underneath which the lifting hydrofoils are attached by struts. When such a boat moves at a sufficiently high speed, the hull is lifted clear of the water surface by the hydrodynamic lift on the underwater foils, and the resistance is thereby considerably reduced. Consequently, after it is foil-borne, a hydrofoil boat could travel much faster than a boat of conventional type with the same propulsive power. While planing boats also have comparatively low resistance at high speeds, the hydrofoil craft appears to have some distinct advantages, such as a higher transport efficiency* and smoother running especially in choppy water.[1] Thus, the use of hydrofoils in many applications would undoubtedly result in important savings in power and time.

Before the advantages of hydrofoil craft can be exploited to their fullest extent, however, it is first necessary that certain difficulties standing in the way of further development be removed. In Ref. 1, an expository investigation has been given which points out some of the existing gaps in fundamental knowledge about hydrofoil craft. Some of these problems which must be solved if the benefits of hydrofoil operation are to be realized are worth mentioning here: for instance, (1) the stability problem, especially in a seaway, (2) the design of a satisfactory power plant and propulsion system, and (3) the problem of hydrodynamic performance when cavitation occurs. The present report is concerned only with this last problem.

The importance of cavitation in hydrofoil operation can be seen as follows. The advantages of such hydrofoil craft become more prominent at higher speeds (cf. Fig. 13 in Ref. 1 for this criterion), but the problem of cavitation appears when the speed is sufficiently high. For a

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*Here the "transport efficiency" is defined, following Ref. 1, to be the total load multiplied by speed and divided by driving power.

given geometrical profile of the hydrofoil, there is a critical speed at which the cavity wake starts to form. For speeds higher than this critical value, the cavity is well-developed, and thus it seems that operation under cavitating conditions is almost inevitable in the future practical applications of hydrofoils. This problem becomes even more important when the design specification calls for long-duration cruising under cavitating conditions. Thus it seems clear that a basic investigation of the hydrodynamic characteristics of fully cavitating hydrofoils certainly deserves some attention.

It is the purpose of this report to compare in a rather idealized way the different properties of fully cavitating and fully wetted hydrofoils. A recent theoretical investigation, together with some experimental data,[3] throws more light on the understanding of the two-dimensional hydrofoil problem when the cavity flow is fully developed, and the results obtained there will be used as a basis for the present discussion. The effects of finite span and the free surface are also commented upon and some estimates are made of the drag of the supporting struts and appendages under cavitating operation. Finally, a few remarks are given on some practical aspects of the hydrodynamical problem. It is hoped that some of the essential differences, advantages, and disadvantages of operating hydrofoils in the cavitating and noncavitating regimes will thereby be made clear.

II. STATEMENT OF THE PROBLEM

In order to provide background for subsequent discussion, some of the salient facts about cavitation and air ventilation will be summarized here.[17] Cavitation is a dynamic process involving the formation and collapse of vapor-filled cavities (or perhaps cavities filled with a vapor-gas mixture) in a flowing liquid. With a normal liquid flowing over a solid body, these cavities form in regions where the local pressure drops to vapor pressure and would drop below vapor pressure if the cavity did not form. Conversely, a cavity starts to collapse when the vapor cavity is transported or extended into a region where the local pressure is above vapor pressure. In other words, cavitation is actually the local boiling of the liquid due to reduced static pressure. Observations have shown that there are several different types of cavitation. (1) In incipient cavitation, that is, in an early stage of cavitation under development, a series of discrete small vapor bubbles are formed. Most of them travel with the liquid, but some have the tendency to stick to the solid surface. These bubbles continue to grow as long as the pressure is at or below the vapor pressure and start to collapse as the pressure becomes greater than the vapor pressure. In this stage the entire flow field is essentially the same as for noncavitating flow. (2) In partial (or limited) cavitation, which would result if the flow velocity were gradually increased, the vapor bubbles coalesce and form a more or less integral vapor cavity, covering only a part of the solid surface. In this stage the single cavity remains attached to the solid surface but, otherwise, has not grown to extend into the rear of the body. The entire flow field is, therefore, still much the same as before. The streamlines are displaced slightly away from the body due to the existence of the thin bubble layer. It has been observed that the bubble layer lessens the surface friction by acting as a cushion between the solid surface and the high velocity water stream. (3) If the flow velocity is further increased, the integral cavity grows larger and extends well out into the flow; the higher the speed, the larger the size of the cavity. Inside the cavity, the pressure of the vapor, or vapor-gas mixture, is almost constant. The flow outside the cavity differs to a great extent from the noncavitating flow. This latter stage is usually referred to as the fully cavitating flow regime, and is the one which will be given most consideration in the following discussion.

An air cavity can also be initiated by permitting air to ventilate into a vapor cavity which has already formed. For example, if a vapor cavity has formed on a body running shallowly below the water surface, the air can be introduced from the atmosphere to the cavity by some means, for instance, a pipe connecting these two regions. The effect of this is to increase the cavity pressure, and hence to change the forces on the body and the size of the cavity. If the air flow is sufficient, it will equalize the pressure between the surface and cavity so that the cavity pressure will be atmospheric, corresponding to the highest attainable value of the cavity pressure. In general, the cavity pressure (of a gas-vapor mixture) will be intermediate between this upper limit (atmospheric) and the vapor pressure.

The general character of the flow with full cavity is independent of the constituents of the cavity gas but depends on the value of the cavitation number σ defined by

$$\sigma = \frac{p_\infty - p_\sigma}{\frac{1}{2}\rho U^2} \tag{1}$$

where p_σ is the pressure in the cavity, p_∞ and U are the fluid pressure and velocity at infinity, and ρ is the liquid density. Thus, the cavitation number is simply the negative of the pressure coefficient as it would usually be defined for the cavity pressure. If the effect of gravity is appreciable, the pressure p_∞ may vary considerably from top to bottom of the cavity so that a given body does not have, in a strict sense, a fixed cavitation number. In practice, however, this effect is quite small since in most cases to be considered the Froude number, as defined by $Fr = U^2/g\ell$ where ℓ is the vertical characteristic dimension of the body, is very large and the gravity effect is then negligible. In these cases, it is sufficient to refer p_∞ to its mean value taken vertically across the cavity.

In the design of hydrofoils where cavitation is a problem, it is known experimentally that as cavitation develops the lift decreases while the drag increases, a behavior which is undesirable from the efficiency viewpoint. Consequently, certain efforts have been made to shape the various components (such as the hydrofoil, supporting struts, etc.) in such a way that cavitation could be avoided within the required speed range. Thus the lifting hydrofoil and supporting struts are made comparatively thin, and, probably even more important, the camber of the hydrofoil is made small so that the local static pressure will not be as low as the vapor pressure. However, even for a system of well-designed components, there exists an upper limit of flow velocity above which cavitation will take place. On the other hand, for a moderately high design speed, the techniques to ascertain the highest cavitation speed would be hindered by several practical limitations. For instance, the requirement of structural sturdiness is one; and second, a limitation of hydrodynamical nature that the necessary total lift, when the velocity is limited to be less than the cavitation speed, can only be obtained by increasing the lifting area, and hence also the frictional drag. In order to achieve a thorough clarification of the problem as to whether optimum conditions would be obtained by designing for the noncavitating or cavitating motion, it seems constructive to first compare the characteristics of hydrofoils under cavitating and noncavitating operations. In the next section, the comparison is initiated with a study of two-dimensional hydrofoils.

III. TWO-DIMENSIONAL CAVITATING AND NONCAVITATING HYDROFOILS

As a preliminary investigation, we first consider the hydrodynamic behavior of hydrofoils of infinite span in a flow of infinite extent under two different operating conditions: fully cavitating and fully wetted. In order to simplify the comparison, we shall confine the discussion to two geometric shapes, namely, the flat plate and circular arcs, the results of which are more or less representative of hydrofoils of other geometrical configurations. The effect of thickness will be neglected in this report, as this effect is rather unimportant for thin hydrofoils.

In some of the previous works,[4, 5] the lift coefficient of fully cavitating hydrofoils at a small angle of attack α and arbitrary cavitation number σ is approximated by the expression

$$C_L = \frac{\pi}{2}\alpha + \sigma, \tag{2}$$

where σ is defined by Eq. (1). Formula (2) was deduced by Betz[4] from Rayleigh's theory of the oblique lamina (e.g. Ref. 6, p. 102) by linearizing the lift coefficient at $\sigma = 0$,

$$C_L = \frac{2\pi \sin\alpha}{4 + \pi \sin\alpha}, \tag{3}$$

for α small to obtain $\pi/2\ \alpha$ and by adding to this quantity the pressure coefficient on the upper suction side, namely, σ. As will be shown later, the agreement between the linearized formula of Eq. (2) and experiment is not very good, even for α small (see Ref. 7). Furthermore, the effect of the camber of cavitating hydrofoils is important in applications and such a correction is not indicated in Eq. (2).

Recently a theoretical investigation, supported by experimental evidence, has been carried out,[3] in which the free streamline theory is applied to study the lifting problem of two-dimensional hydrofoils with a fully cavitating wake. The hydrofoils considered in Ref. 3 are those with a sharp leading and trailing edge which are assumed to be the separation points of the free streamlines. Except for this limitation, this nonlinear theory is applicable to hydrofoils of any geometric profile, operating at any cavitation number and for almost all angles of attack as long as the wake has a fully cavitating configuration. Two examples are worked out in Ref. 3 for the case of (1) the flat plate, and (2) the circular arc, subtending an angle 2γ, as shown in Fig. 1.

The result for the circular arc is as follows:

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 c} = \frac{2\pi}{J}\left(1+\sigma+\frac{\epsilon^2}{4}+\frac{\gamma^2}{6}\right)\left\{\left[\sin\beta\cos\beta + A_1\cos\beta + \frac{A_2}{2}\right]\right.$$

$$\left. + \frac{\epsilon^2}{4}\frac{\cos\beta}{(\sin\beta + A_1/2)^4}\left[\left(\sin\beta+\frac{A_1}{2}\right)^3 + \sin\beta\left(\sin\beta+\frac{A_1}{4}\right)\left(\frac{A_1}{2}+\frac{5}{4}A_2+A_3\right)\right]\right\}; \quad (4)$$

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 c} = \frac{2\pi}{J}\left(1+\sigma+\frac{\epsilon^2}{6}+\frac{\gamma^2}{6}\right)\left[\sin\beta + \frac{A_1}{2}\right]^2, \quad (5)$$

where

$$J = 4 + \pi\sin\beta + A_1\left(\pi + \frac{8}{3}\sin\beta\right) + \frac{\pi}{2}A_2\cos\beta - \frac{8}{15}A_3\sin\beta,$$

$$\beta = \alpha + \frac{1}{2}\delta - \frac{\epsilon^2}{4(\pi-\alpha+\gamma)} - \frac{\gamma}{4+\pi\sin\alpha}\left\{\frac{\alpha+\gamma}{[(\alpha+\gamma)^2+\epsilon^2]^{1/2}}\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}\right\},$$

$$\epsilon = \frac{1}{2}\log(1+\sigma), \qquad \delta = \left[(\alpha+\gamma)^2+\epsilon^2\right]^{1/2} - (\alpha+\gamma),$$

$$A_1 = \gamma\left[1+\frac{\pi\sin\alpha}{8(4+\pi\sin\alpha)}\right] + \frac{9}{16}\delta\left\{1+\frac{\gamma(1+\cos\alpha)}{9[(\alpha+\gamma)^2+\epsilon^2]^{1/2}}\right\} + \frac{9\epsilon^2}{32(\pi-\alpha+\gamma)},$$

$$A_2 = \frac{\gamma}{4+\pi(A_1+\sin\beta)}\left\{\frac{\alpha+\gamma}{[(\alpha+\gamma)^2+\epsilon^2]^{1/2}}\cos^2\frac{\beta}{2} - \sin^2\frac{\beta}{2}\right\},$$

$$A_3 = \frac{1}{9}(\gamma - A_1) - \frac{\gamma}{9(4+\pi\sin\alpha)}\left\{\pi\sin\beta + \frac{2\delta(1+\cos\alpha)}{[(\alpha+\gamma)^2+\epsilon^2]^{1/2}}\right\}.$$

The result for flat plate can be deduced from the above formulas by letting $\gamma = 0$. The value of C_L for the flat plate is plotted against α for different σ's in Figs. 2 and 3 and is further cross-plotted against σ in Fig. 6. The value of C_D is similarly plotted in Figs. 4, 5, 7. In the cavitating range of practical interest, the Reynolds number of the flow is in general very large, say, of the order 5×10^5 or greater. The frictional drag coefficient is then of the order 0.005 (see Eq. (8)) which is much smaller than the cavity drag coefficient for almost all α and σ (cf. Figs. 4, 5) and can thus be neglected. A series of experiments[7] has been carried out in the Hydrodynamics Laboratory, California Institute of Technology, as part of this program. Some of the experimental data are shown in Figs. 6 and 7, with which the theory is in good agreement.

In order to compare some hydrodynamic characteristics of fully cavitating hydrofoil with those at fully wetted condition, we recall the well-known result[8] from airfoil theory that, when the hydrofoil is fully wetted,

$$C_L = 2\pi \sin(\alpha + \frac{\gamma}{2}) , \tag{6}$$

$$C_D = (2 \cos \alpha) C_f \tag{7}$$

where C_f, the mean friction coefficient on one side of the wing, can be estimated by using the Prandtl-Schlichting formula (see Ref. 9, p. 33)

$$C_f = 0.455(\log_{10} \mathrm{Re})^{-2.58}, \qquad \text{for} \qquad 10^6 < \mathrm{Re} = \frac{Uc}{\nu} < 10^9 . \tag{8}$$

The above equation gives an approximate value of C_f for a flat plate at zero angle of attack with a turbulent boundary layer and should provide a good approximation for the present comparison. In the experimental work of Ref. 7, the measured Re falls in the above range. The value of C_L given by Eq. (6) is plotted in Figs. 2 and 3 for flat plate and in Fig. 8 for a circular arc with $\gamma = 8^\circ$.

Let us take Fig. 3 as a typical illustration. For a given value of σ there is a certain small positive value of α, say α_p, at which the

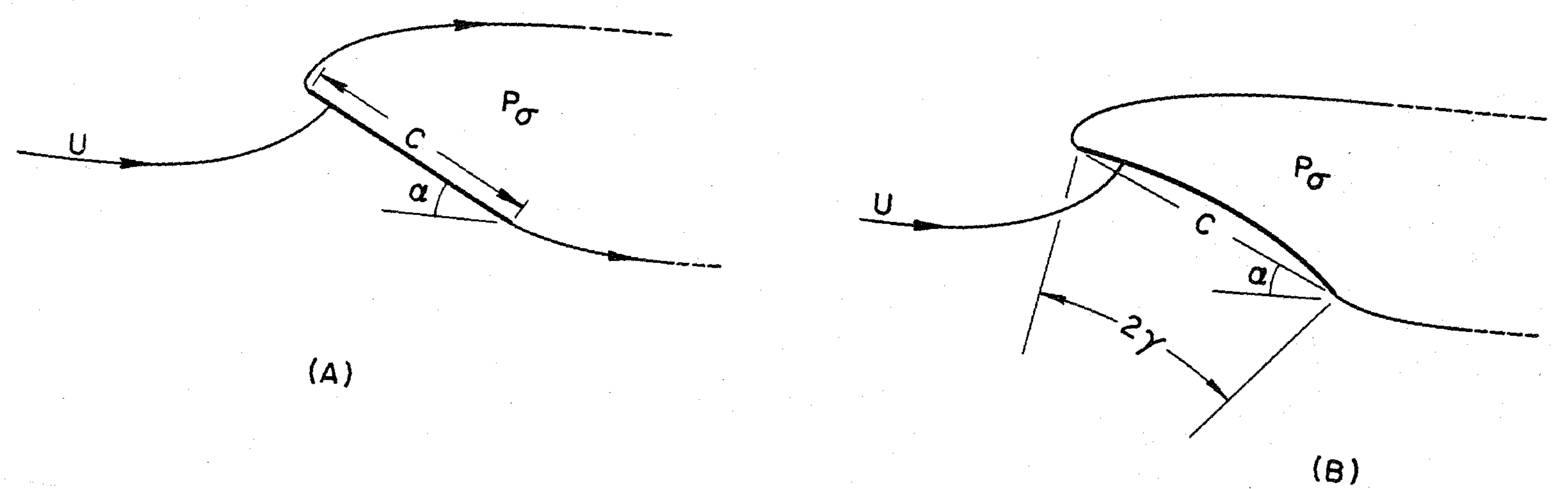

Fig. 1 - Two configurations of fully cavitating hydrofoils.

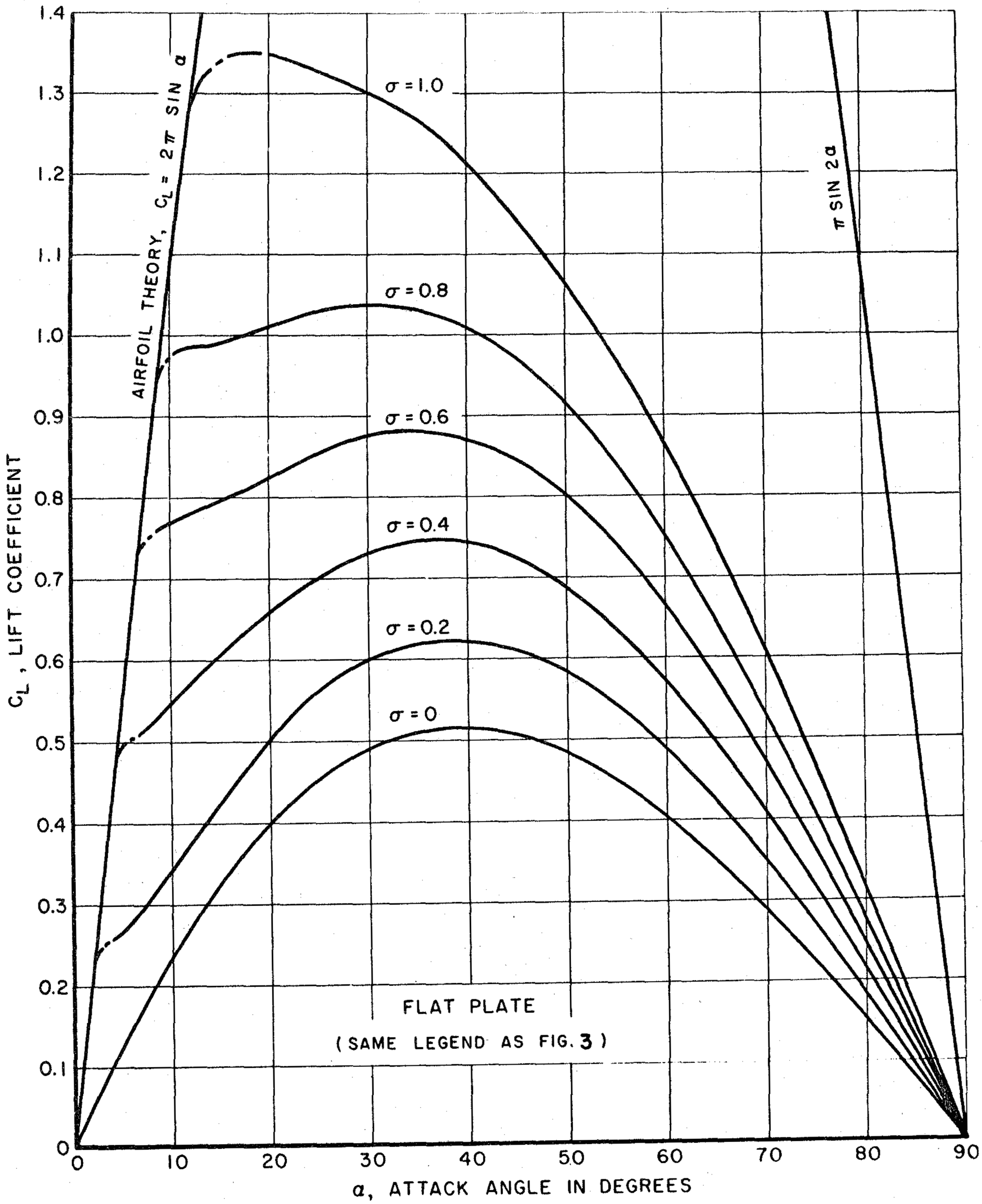

Fig. 2 - Dependence of C_L on a.

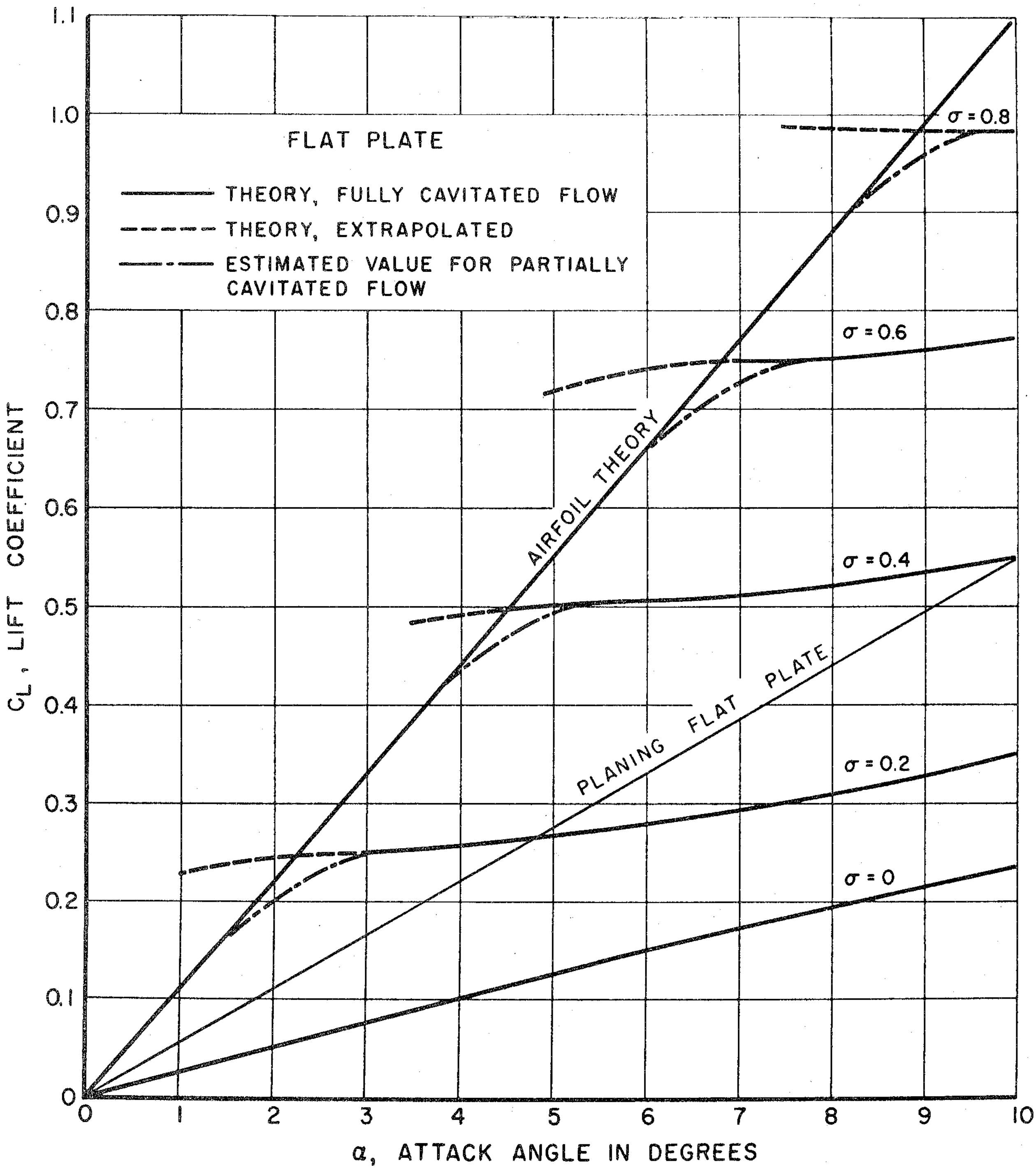

Fig. 3 - The value of C_L for α small.

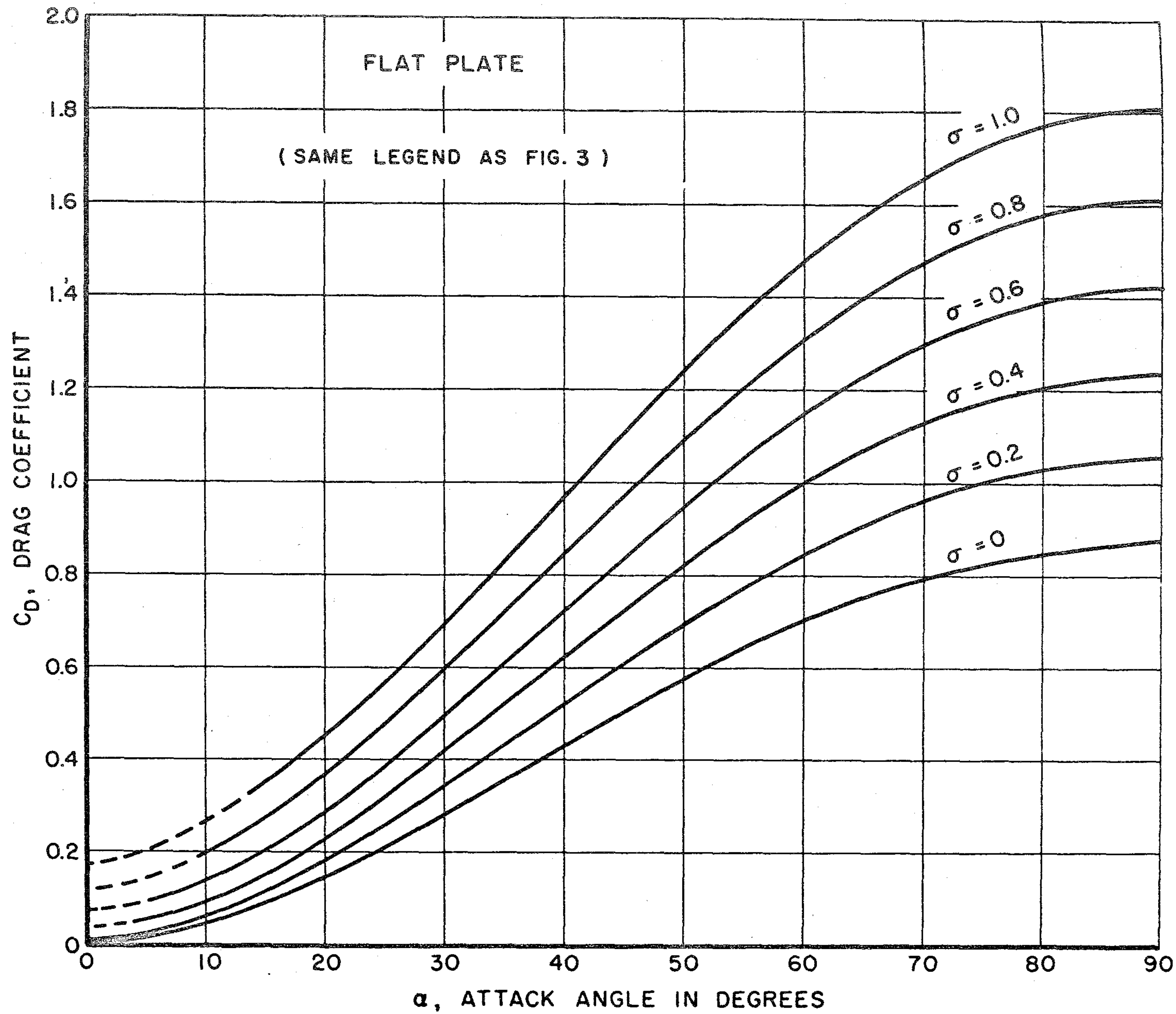

Fig. 4 - Dependence of C_D on α.

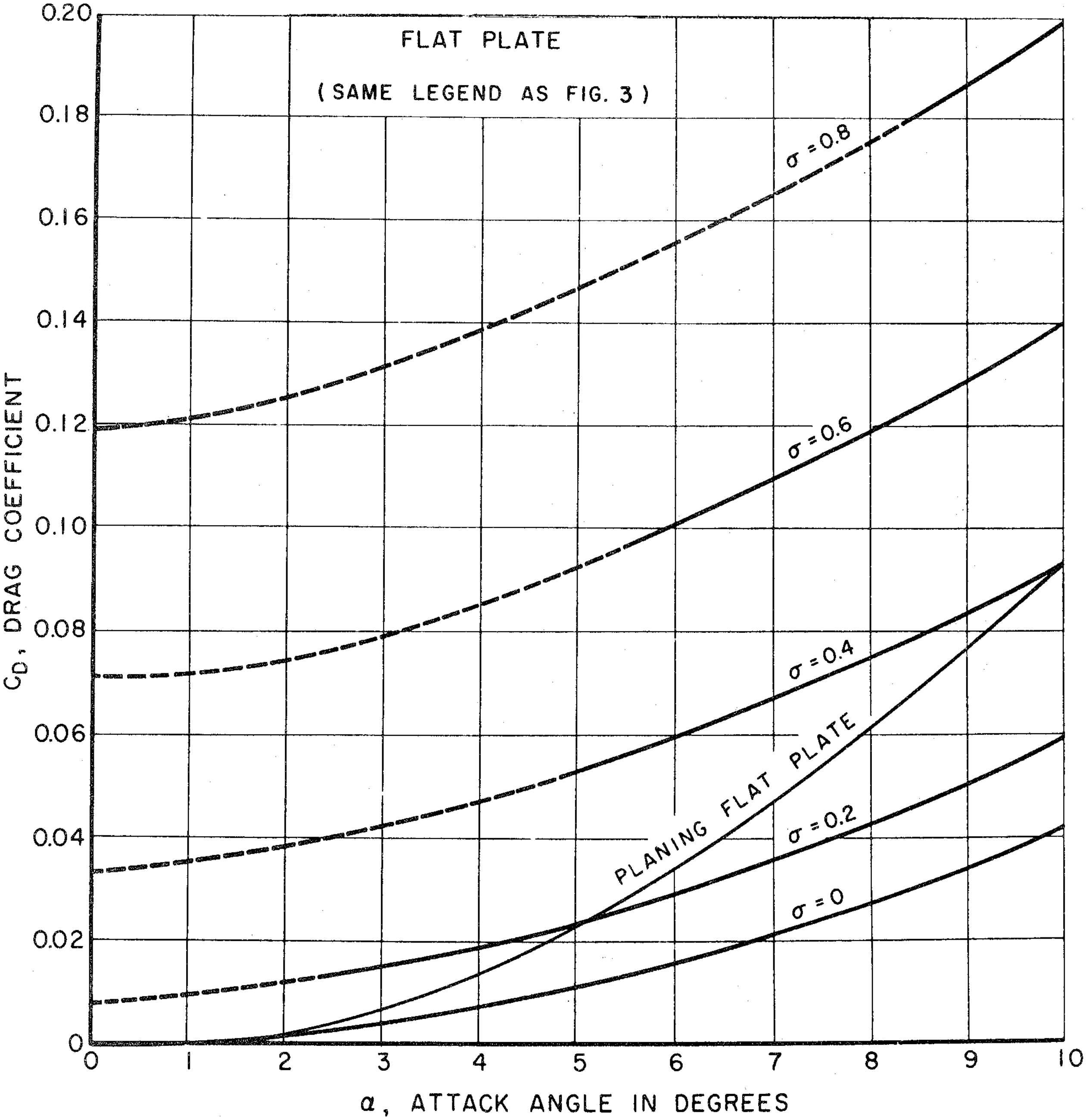

Fig. 5 - The value of C_D for α small.

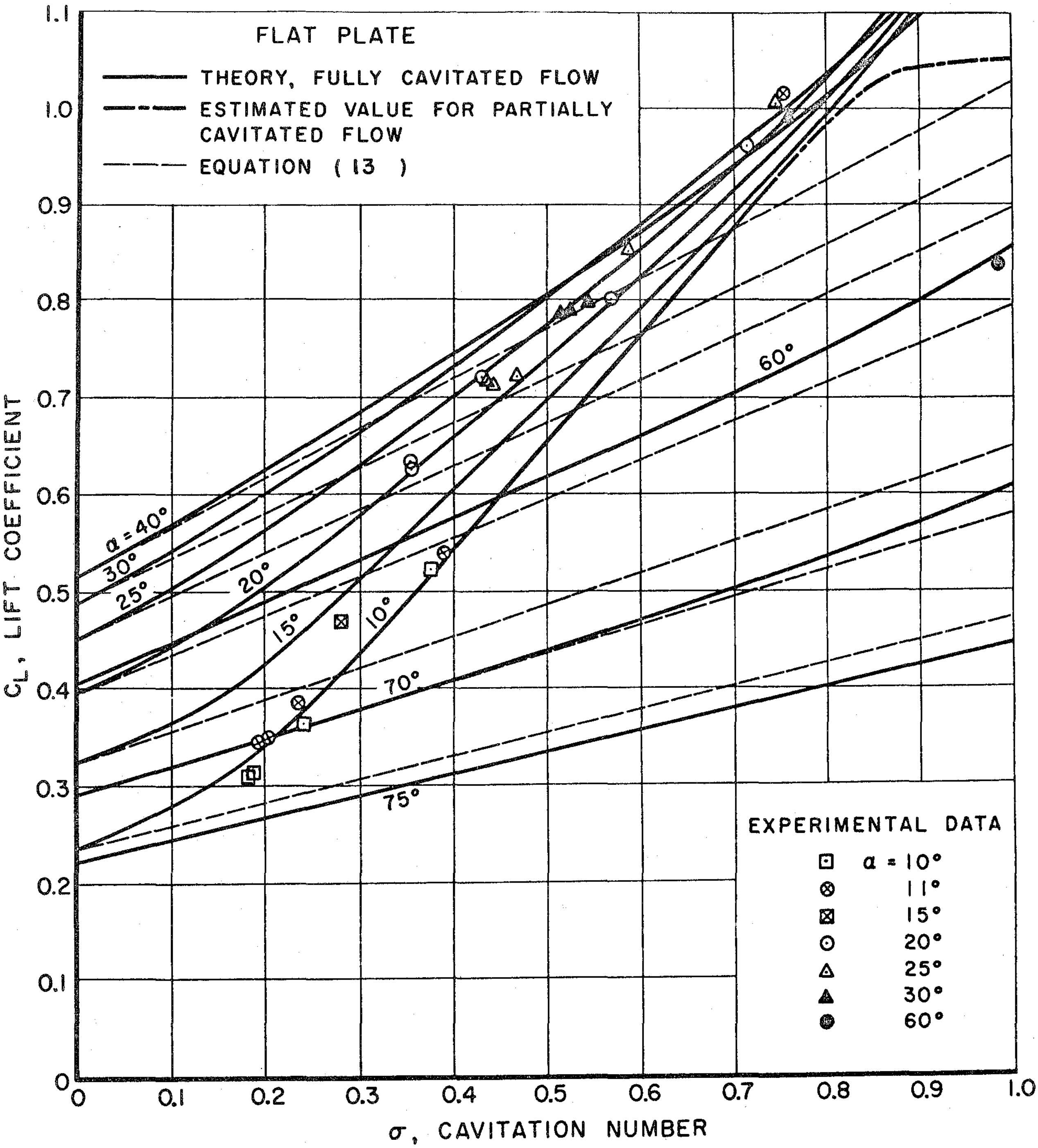

Fig. 6 - Dependence of C_L on σ.

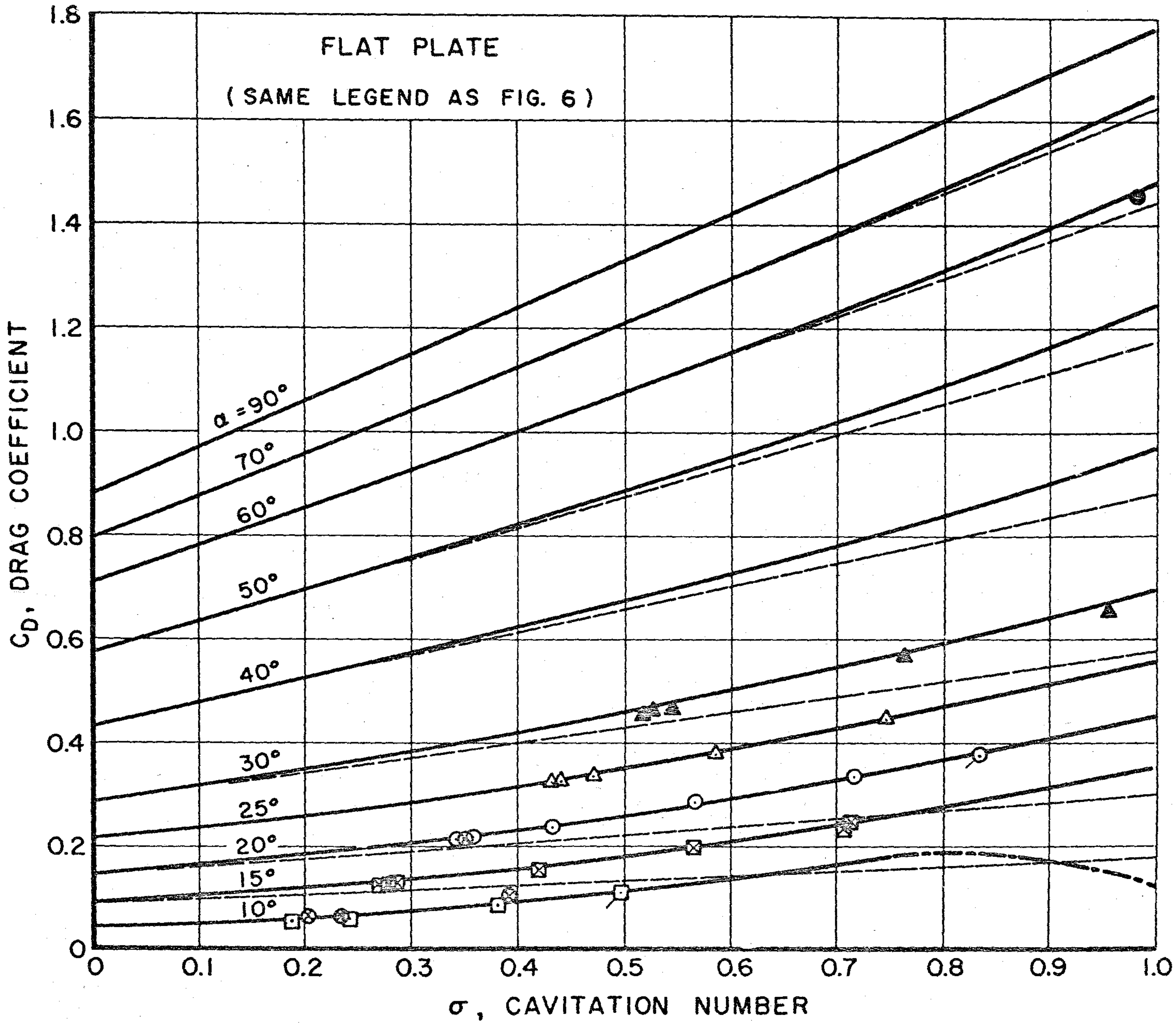

Fig. 7 - Dependence of C_D on σ.

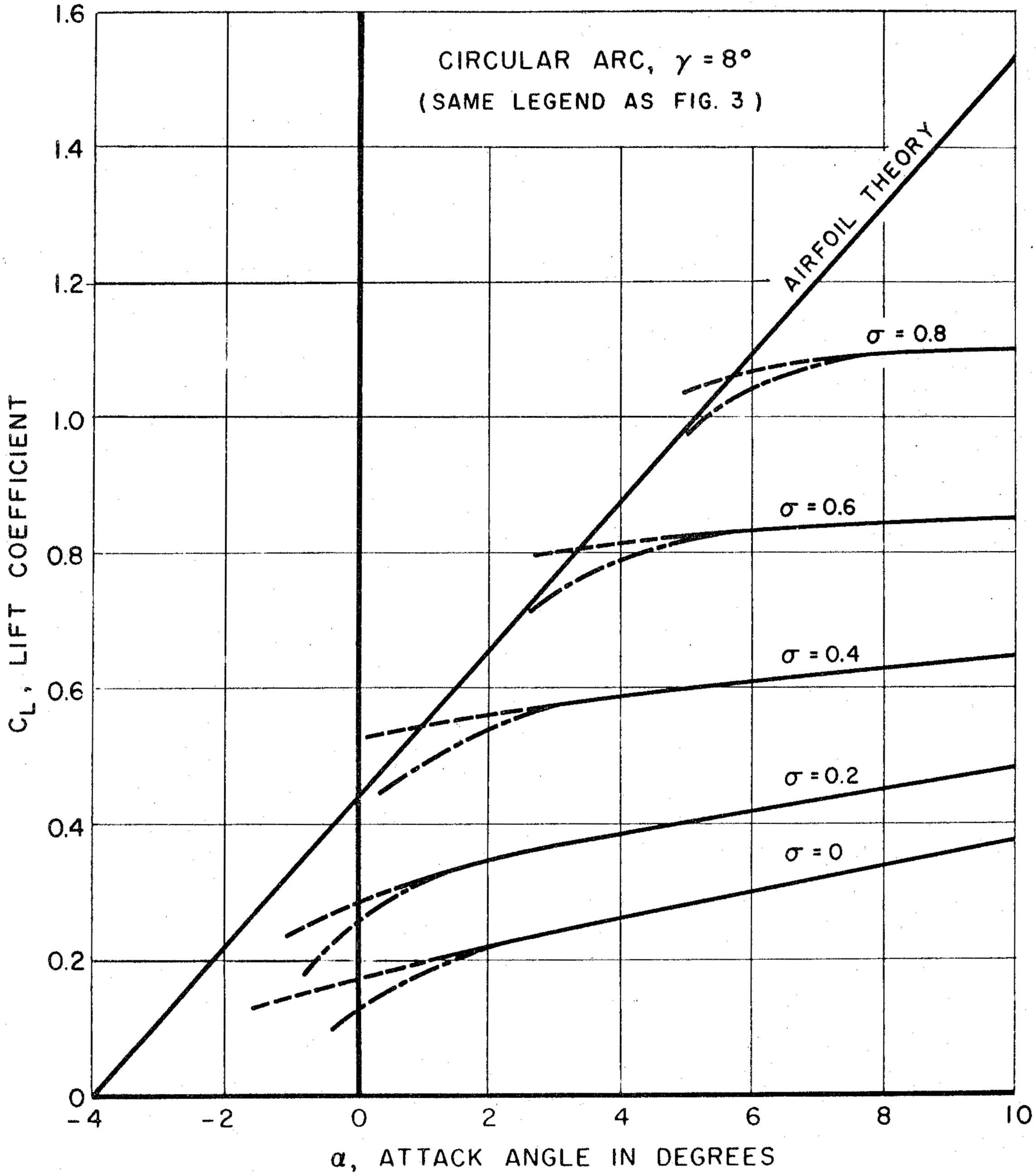

Fig. 8 - The value of C_L of a circular arc.

cavitating value of C_L, given by Eq. (4), becomes equal to the aerodynamic value given by Eq. (6), provided that the fully cavitating model is still assumed to be a possible configuration (e.g. $\alpha_p = 4.5^o$ for $\sigma = 0.4$). Any further extrapolation of Eq. (4), without justification, to smaller values of α would yield the implausible result that the cavitating values of C_L would be greater than their corresponding aerodynamic values. It seems reasonable, however, that under no cavitating condition should the value of C_L surpass noticeably its corresponding aerodynamic value. Thus we expect that near $\alpha = \alpha_p$ the partially cavitating flow takes place, a transitional stage between the fully cavitating and fully wetted conditions. This argument is supported by experimental evidence, as shown by the double-dotted lines in Figs. 2, 3. Thus, the value of C_L of a fully wetted hydrofoil is actually the asymptote to which the cavitating C_L at every σ approaches from below as α decreases from α_p. After the cavity is developed, C_L is less than the aerodynamic value at the same α for $\alpha_p < \alpha < \pi/2$, and falls farther below the aerodynamic value the smaller σ, (see Fig. 2). Furthermore, the cavitating drag coefficient increases to values much larger than those under the fully wetted condition. Therefore, if the ratio C_L/C_D is the quantity to be compared, the fully cavitating hydrofoils should always have smaller lift-drag ratio than those operating under the fully wetted condition.[3] Even under the unfavorable circumstance that cavitation cannot be avoided, there are still several interesting points worth noting:

a. The effect of camber of the geometric profile.

Under the fully wetted condition, the rate of increase of C_L with respect to γ for α and γ both small can be deduced from Eq. (6);

$$\left(\frac{d\,C_L}{d\gamma}\right)_{\alpha=0,\ \gamma=0} = \pi \; . \tag{9}$$

This rate for a fully cavitating hydrofoil can be determined[3] from Eq. (4) to be

$$\left(\frac{d\,C_L}{d\gamma}\right)_{\alpha=0,\ \gamma=0} = \frac{7}{16}\pi \tag{10}$$

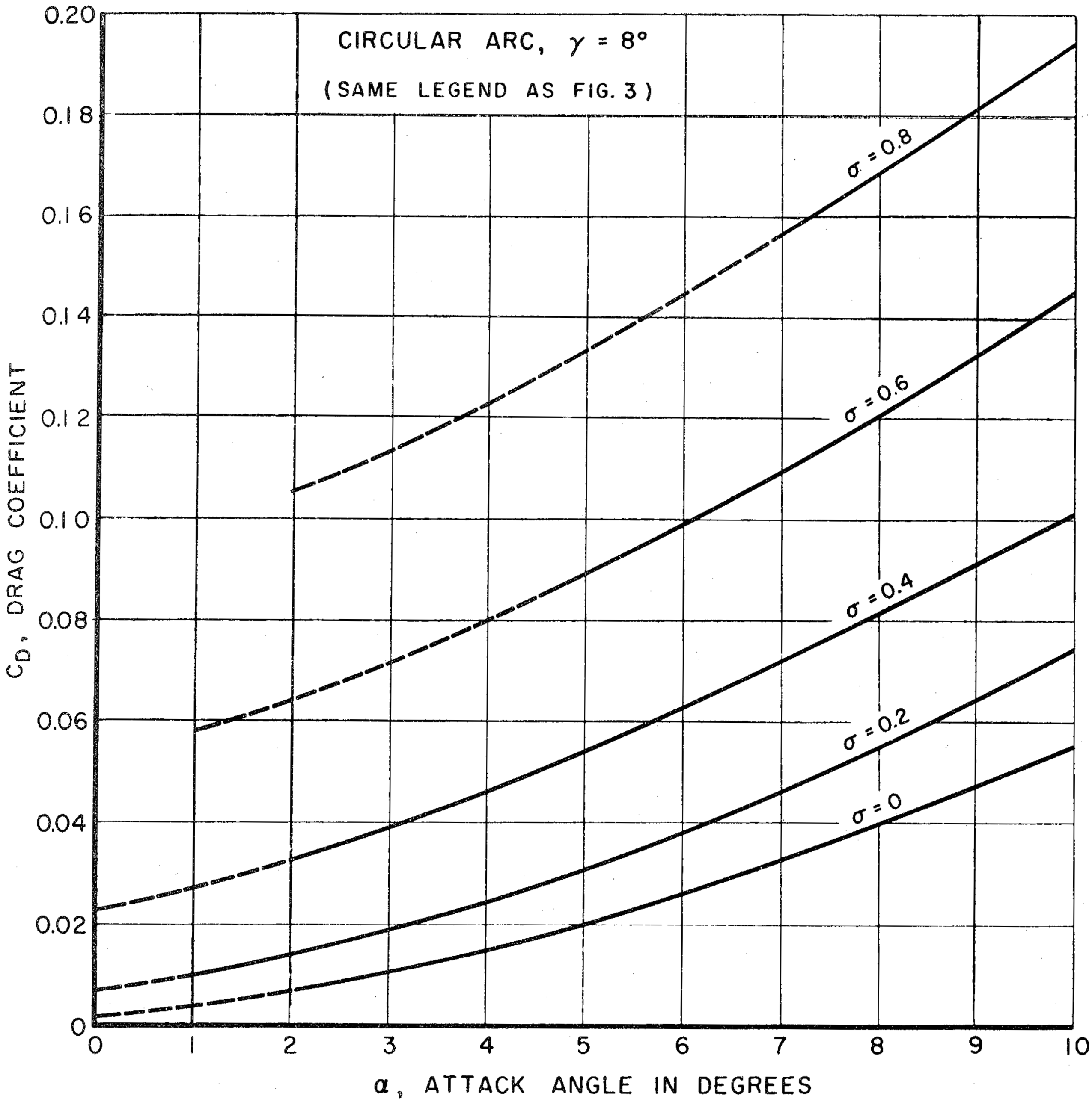

Fig. 9 - The value of C_D of a circular arc.

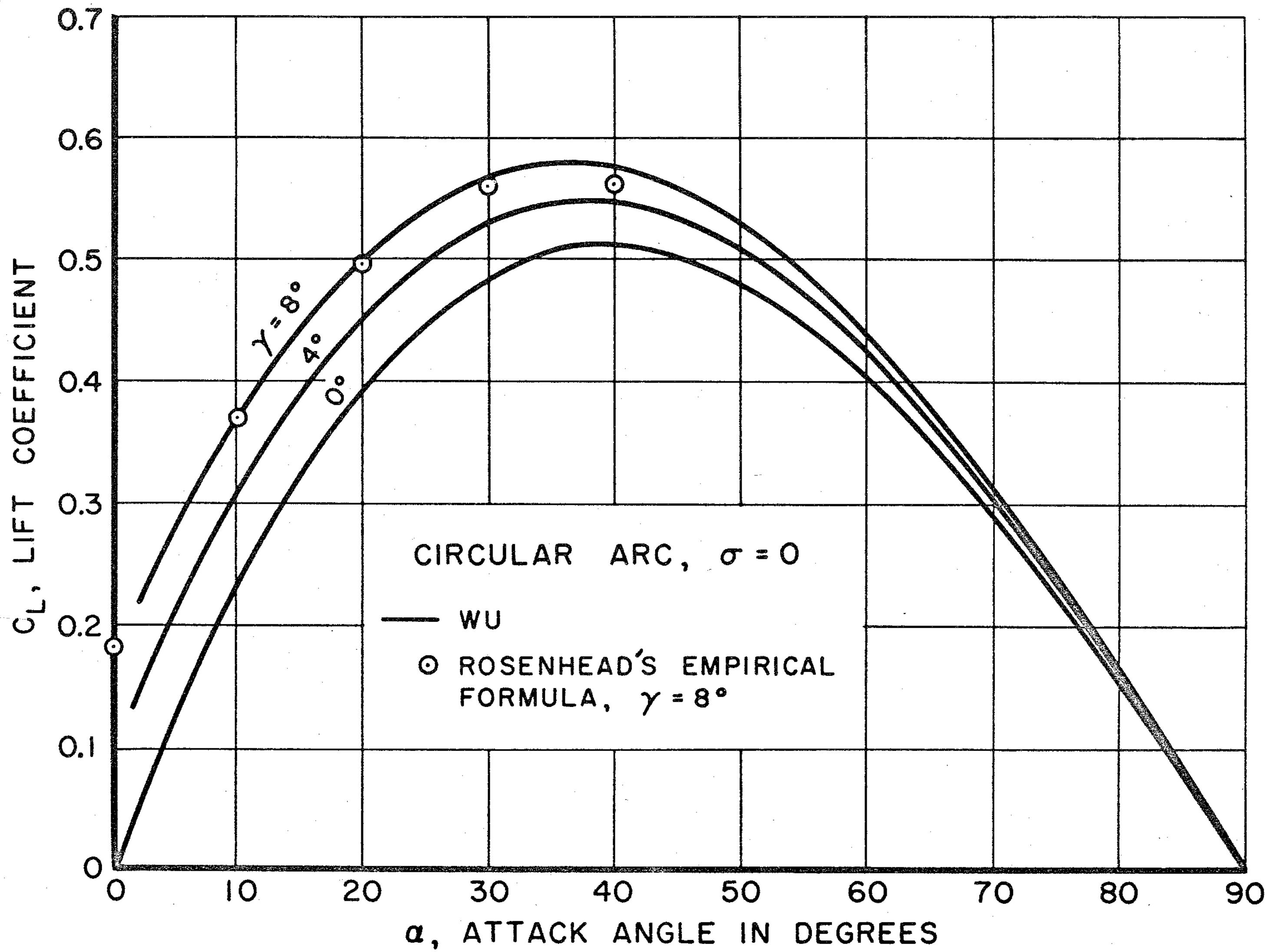

Fig. 10 - The effect of camber on C_L at $\sigma = 0$.

which is insensitive to σ for $\sigma < 0.8$. Although the above value of $(dC_L/d\gamma)$ for cavitating flow is about 56% less than that of the fully wetted case on their respective absolute scale, the comparison can be made on the basis of equal aerodynamic effective angle of attack. The effective angle of attack for the circular arc is known[8] from airfoil theory to be

$$\alpha_e = \alpha + \frac{\gamma}{2} , \tag{11}$$

where α is the geometric angle of attack measured from the chord. At the same α_e the aerodynamic value of C_L is the same for all γ when α_e is small. But if the hydrofoil is fully cavitating, C_L still increases with increasing γ , holding α_e fixed. The rate of increase in this case is approximately:

$$\left(\frac{d\, C_L}{d\gamma}\right)_{\gamma = 0,\ \sigma = 0} \cong 0.5 \tag{12}$$

at equal, small α_e.

The above result shows that the means of increasing C_L by using hydrofoils with larger camber is of greater value in cavitating operation than in the fully wetted condition. In other words, two hydrofoils with different camber, when fully wetted, have the same C_L at the same α_e; after the cavity flow is fully established, the one with the larger camber experiences more lift, holding both α_e and σ the same. When the cavitating case alone is considered (cf. Figs. 10, 11), the situation becomes even clearer: The increase in C_L due to an increment of γ is significant, especially for small α. Besides, the penalty of causing higher drag for larger γ is much less significant, relative to the gain in lift, for α small (cf. Fig. 11).

In view of the above discussion, it is of advantage to use reasonably large camber to obtain higher lift in the fully cavitating regime.

b. The effect of cavitation number.

The results plotted in Figs. 6 and 7 show that for α large, say greater than 45°, the values of C_L and C_D for different σ approach

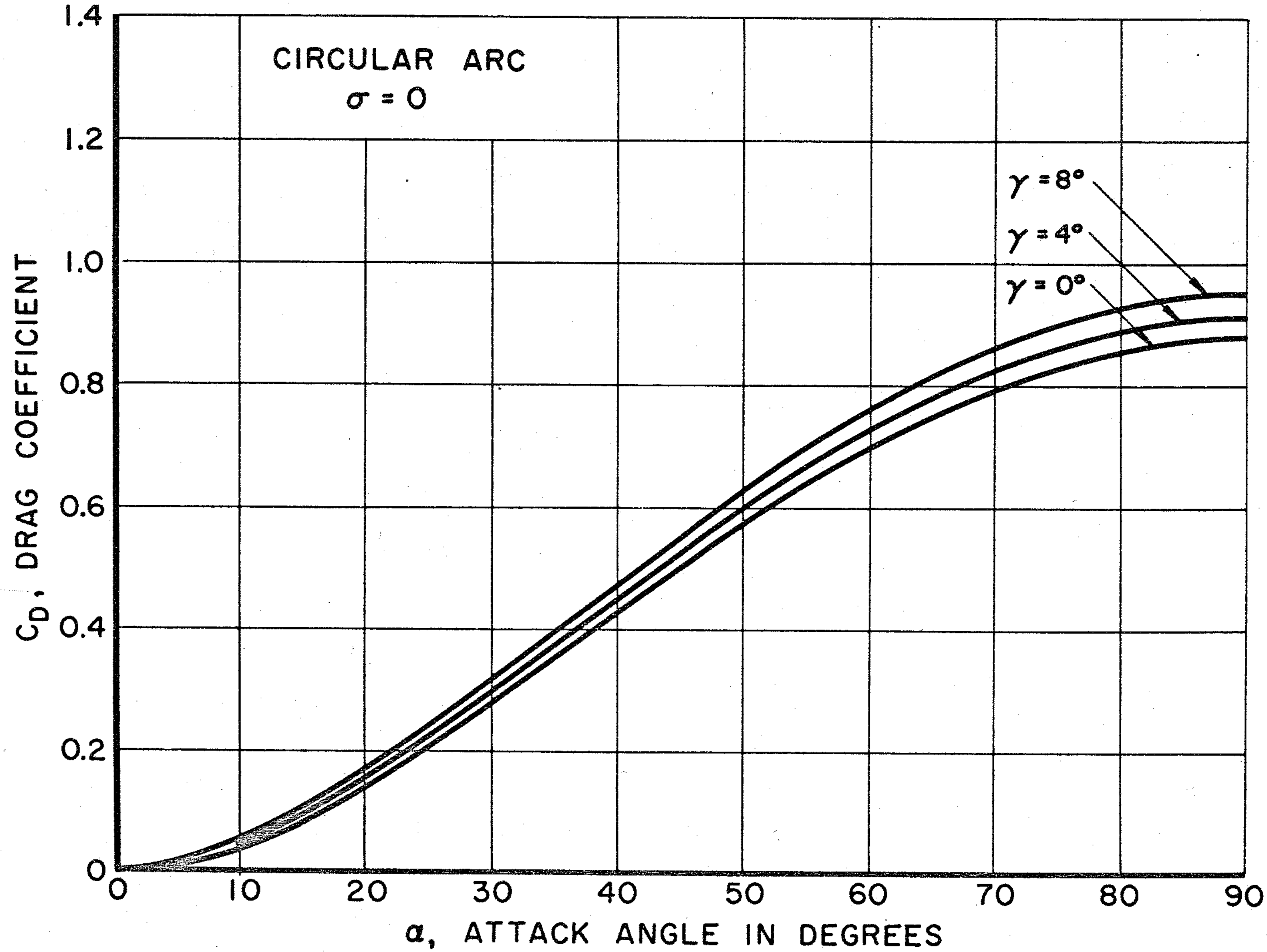

Fig. 11 - The effect of camber on C_D at $\sigma = 0$.

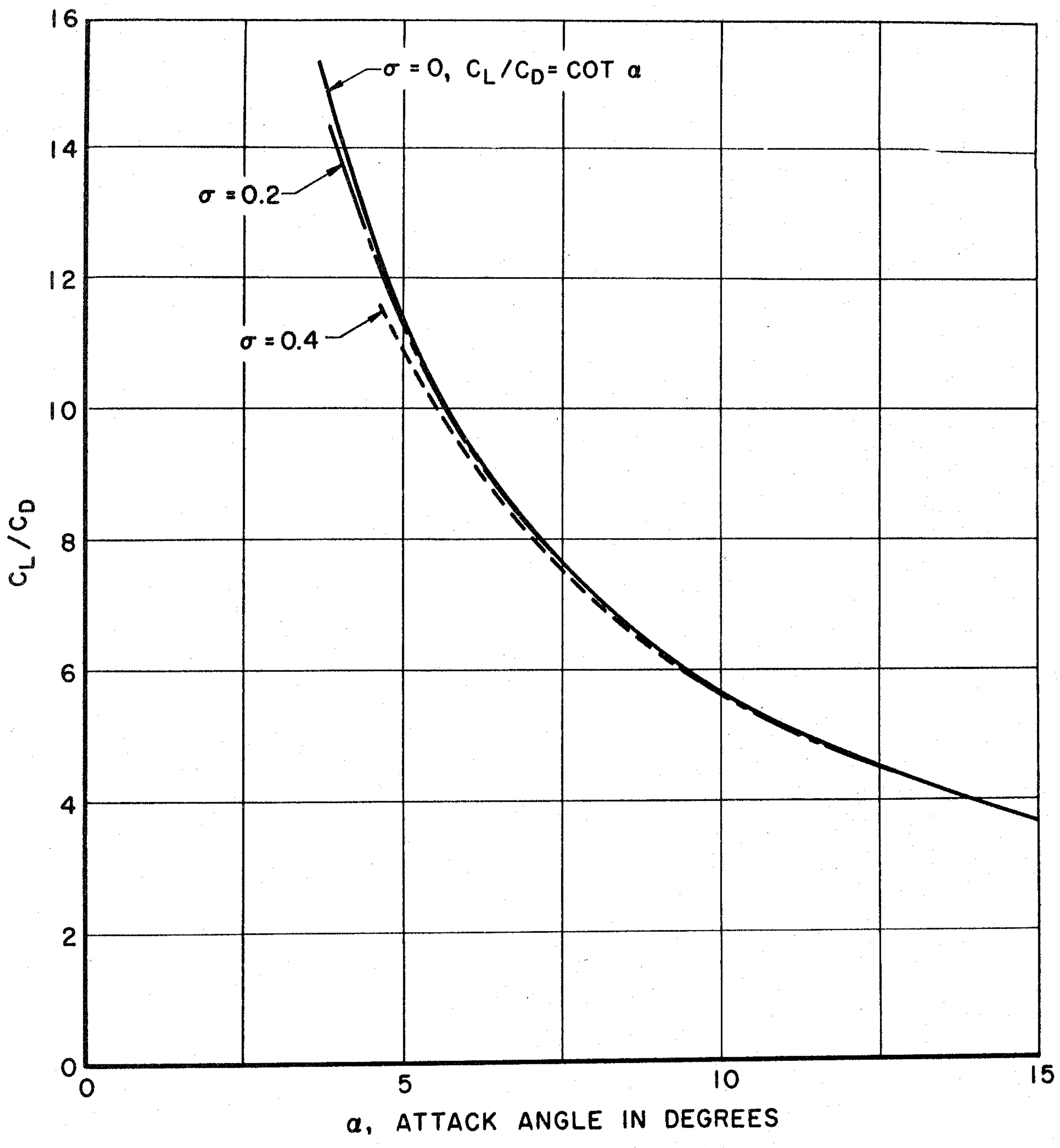

Fig. 12 - The effect of α on C_L/C_D for the flat plate.

respectively the asymptotes

$$C_L(\sigma, \alpha) = (1 + \sigma)\, C_L(0, \alpha)\,, \tag{13a}$$

$$C_D(\sigma, \alpha) = (1 + \sigma)\, C_D(0, \alpha)\,. \tag{13b}$$

In particular, for the flat plate held at $\alpha = 90^\circ$, Eqs. (4), (5) reduce to the well-known results (see Ref. 6, p. 101)

$$C_D(\sigma) = \frac{2\pi}{\pi + 4}(1 + \sigma) = 0.880\,(1 + \sigma)\,, \quad C_L = 0\,. \tag{14}$$

For α small, they deviate appreciably from these asymptotes; the deviation is much more marked for C_L. For example, $dC_L/d\sigma$ becomes greater than unity for $\alpha < 15^\circ$ and $\sigma > 0.2$. Thus, for still smaller α, which is actually in the range of practical interest, the larger value of $dC_L/d\sigma$ favors the operation of hydrofoils at higher σ, that is, at lower velocity U and/or at higher $(p_\infty - p_\sigma)$. From a practical viewpoint, the requirement on the value of U is usually preassigned in the design. Hence the only alternative left is to increase $(p_\infty - p_\sigma)$. With the presence of a free water surface, this can be achieved by submerging the hydrofoil to greater depths, for the hydrostatic pressure increases with the depth. However, because of structural difficulties in designing long connecting struts, the submergence depth is also limited. But within this limitation, the essential point is that the submergence should not be kept too shallow if a better performance is required at cavitating conditions.

c. The effect of angle of attack.

First, it is noted from Figs. 2, 3, that after the hydrofoil is fully cavitated $dC_L/d\alpha$ decreases to values much smaller than that of the fully wetted case, which is approximately equal to 2π. This result means that the C_L of fully cavitating hydrofoils at small angles of attack is quite insensitive to variations of angle of attack. Hence, when the flow is kept fully cavitating and the hydrofoil is held at an angle of attack larger than α_p, there is little danger of losing the lift due to fluctuations of approaching flow direction. This result clearly indicates another merit of using cavitated hydrofoils. However, it should also be pointed out that with respect to increasing α, C_D increases in percentage more rapidly than C_L under the cavitating condition (cf. also Figs. 5, 6). Their relative rates are plotted in Fig. 12 for the flat plate hydrofoil, which shows that the ratio C_L/C_D decreases with increase in α. Its value should be equal to cot α and independent of σ*; the slight discrepancy shown in Fig. 12 is apparently due to the approximation introduced in the calculation. As the maintenance of lift is of primary concern in practical applications of hydrofoils, reserving additional propulsive power to encounter the possible increase in drag due to variations of α must be regarded as unavoidable sacrifice.

d. The effect of partial cavitation.

Experimentally it was found by Sottorf[10] that partial cavitation on the upper surface of a hydrofoil gives a slightly higher value of the C_L/C_D ratio than the noncavitating case provided that the cavity covers only the front portion of the upper surface. This is probably because a thin cavitation bubble layer reinforces the flow curvature and thereby increases the lift. Besides, the bubble layer lessens the surface friction by acting as a cushion between the solid surface and the high velocity water stream. However, this favorable range of operation is not very stable; and such advantage, as it affords, should not be taken for the following reason.

Let us consider for the moment that a hydrofoil is moving slightly faster than the speed at which the hydrofoil is partially cavitated on the upper surface. Suppose that due to some cause there is a slight change in the flow velocity relative to the hydrofoil. If the velocity U decreases, σ then increases and the cavity is suppressed. The flow approaches the fully wetted configuration; consequently C_L increases and C_D decreases, approaching their respective aerodynamic values. The resulting lower drag in turn would make the hydrofoil accelerate if the

*However, it was observed in the experiments of Ref. 7 that the ratio C_L/C_D of the flat plate in full cavity flows depends to an appreciable extent on the cavitation number for α small (less than 15°), but lies very close to cot α, independent of σ, for larger values of α. The reason for this result, whether it is due to wall effect of the water tunnel or other causes, remains to be determined.

propulsive power were kept unchanged. Moreover, when the free water surface is present, the slightly increased lift would raise the hydrofoil to a shallower depth where the hydrostatic pressure is less. Both of these effects result in a decrease in σ, thereby the cavitation starts to grow again. As a consequence, C_L decreases and C_D increases, thus the value of U is again reduced. This process then repeats itself over and over during which the fluctuation in both lift and drag could be very severe and certainly undesirable.

Perhaps, as an analogy to this critical cavitating condition, the transonic flight of airfoils may be remarked on here, even though their basic features are different. When an airfoil is designed for supersonic flight, the duration of its passage through the sonic velocity should be made as short as possible. Likewise, if a hydrofoil is to be operated with a fully developed cavity, then its motion with velocity very close to critical cavitating value should not be too long.

e. Comparison of hydrofoils with planing surfaces.

As will be made explicit later, the effect of free water surface becomes negligibly small even for moderate submerging depths, say, four chords deep or deeper. Now it would be of interest to estimate the change in C_L and C_D when the hydrofoil rises from a moderate depth to become a planing surface. Wagner's theory for the planing flat plate of infinite span gives the following result[11] for small α:

$$C_L = \pi \sin\alpha \cos\alpha \ , \tag{15}$$

$$C_D = \pi \sin^2\alpha + C_f + C_w \ . \tag{16}$$

In Eq. (16) C_f denotes the friction drag coefficient (see Eq. (8)) and C_w represents the wave drag coefficient due to the gravity waves created by the planing surface. With both C_f and C_w neglected, Eqs. (15) and (16) are plotted in Figs. 3 and 5; the ratio C_L/C_D is also equal to $\cot\alpha$ in this case. However, for α small, both C_L and C_D of a fully cavitating hydrofoil are greater than their planing values for approximately

$$\alpha < 0.5\,\sigma \ . \tag{17}$$

It follows that a cavitating hydrofoil, when held at sufficiently large α and small enough σ, would obtain higher C_L and C_D when it pierces through the water surface, a result which cannot be obtained in the fully wetted motion. This suggests that a fully cavitating hydrofoil operating with $\alpha > 0.5\sigma$ would continue to plane should it broach the water surface. However, a more significant feature is that, for $\alpha < 0.5\sigma$, the loss of lift of a cavitating hydrofoil after planing is in general less than that for a noncavitating hydrofoil. Therefore the motion of a fully cavitating hydrofoil near the water surface very likely would be much smoother than that of a fully wetted hydrofoil, and the tendency to maintain automatically the proper cruising altitude would still be retained if $\alpha < 0.5\sigma$.

f. A numerical estimate of the lift under cavitating condition.

Let us suppose, for example, that a fully cavitating hydrofoil is held at $\alpha = 10^{\circ}$ with $\sigma = 0.2$. Let us further take U to be 110 ft per sec., which corresponds to a submergence depth of about 5 feet with $\sigma = 0.2$. From Fig. 3, the value of C_L at this condition is 0.35. Therefore the total lift per square foot of the hydrofoil is

$$\frac{L}{A} = \frac{1}{2}\rho U^2 C_L = (0.34)\,(110)^2 \text{ lbs/ft}^2 = 2.06 \text{ tons/ft}^2 \;. \qquad (18)$$

The lift force per unit area as large as this magnitude has been observed.[12] Thus, it seems that no particular difficulties are involved in supporting watercraft of medium weight, say, of the order of 100 tons, even if the fully cavitating operation is chosen. The operation of such a craft would be possible provided, of course, the take-off power to have the craft foilborne were available. The present state of the art in applying hydrofoils to heavy ships is primarily limited by the power plants available, as discussed in Ref. 1.

Some of the two-dimensional hydrofoil results given in the preceding sections must of course be corrected to allow for several effects which usually occur in practical applications. Perhaps the most important of these are the effects due to finite span and the presence of free water surface. For noncavitating hydrofoils these problems have received considerable attention and some previous results will therefore be reviewed here. The qualitative features of these effects on cavitating hydrofoils will be discussed subsequently.

IV. THREE-DIMENSIONAL AND SURFACE EFFECTS ON HYDROFOILS

In Ref. 2 a theory for fully wetted hydrofoils of finite span is formulated in a manner similar to the lifting line wing theory of L. Prandtl. In order to discuss the three-dimensional effect of finite span and the gravitational effect (free surface wave formation) we reproduce here the detailed calculation of C_L and C_D for a specified hydrofoil carried out in Ref. 2. The hydrofoil is taken to have an elliptic plan form, with span-chord 5 (and hence aspect ratio 6.3), moving with a geometric angle of attack 6^o at a speed $U = 100$ ft per sec. The result is plotted in Fig. 6 of Ref. 2, in which the viscous drag is neglected. Some experimental results are available.[12] A model used in the tests of Ref. 12 was a NACA 23012, of rectangular plan form with a 5 in. chord and aspect ratio 6, and was operated at speeds above 20 fps. Thus, based on the chord, the Reynolds number Uc/ν is greater than 654,000 and the Froude number $U^2/(gc)$ is greater than 30. Though the special example calculated by applying the theory does not quite correspond to the available data, we can still justify the comparison by noting that first, the effect of these two different plan forms is unimportant and second, the values of $U^2/(gc)$ for the two cases are of the same order of magnitude (38.9 for the theory with chord equal to 8 ft, between 30 and 250 for the experiment) which are sufficiently large to cause no appreciable corrections according to the theory. Besides, the frictional drag and the effect of the different aspect ratio can easily be accounted for. Now we add to the calculated C_D an approximate frictional drag coefficient $C_{Df} = 2C_f = 0.011$, where C_f is estimated by using Eq. (8) for the same Re as the experiment and the factor two accounts for the two sides of the hydrofoil. The final result so obtained is plotted in Fig. 13. In the same figure the NACA data for $\alpha = 6^o$ are also shown for comparison. Thus, one can see that the theory is in good agreement with the experiment.

To summarize, the results show that for Froude number $U^2/(gc)$ greater than 10 and for submergence depths greater than two chord lengths, the influence of the water surface is negligibly small. In the

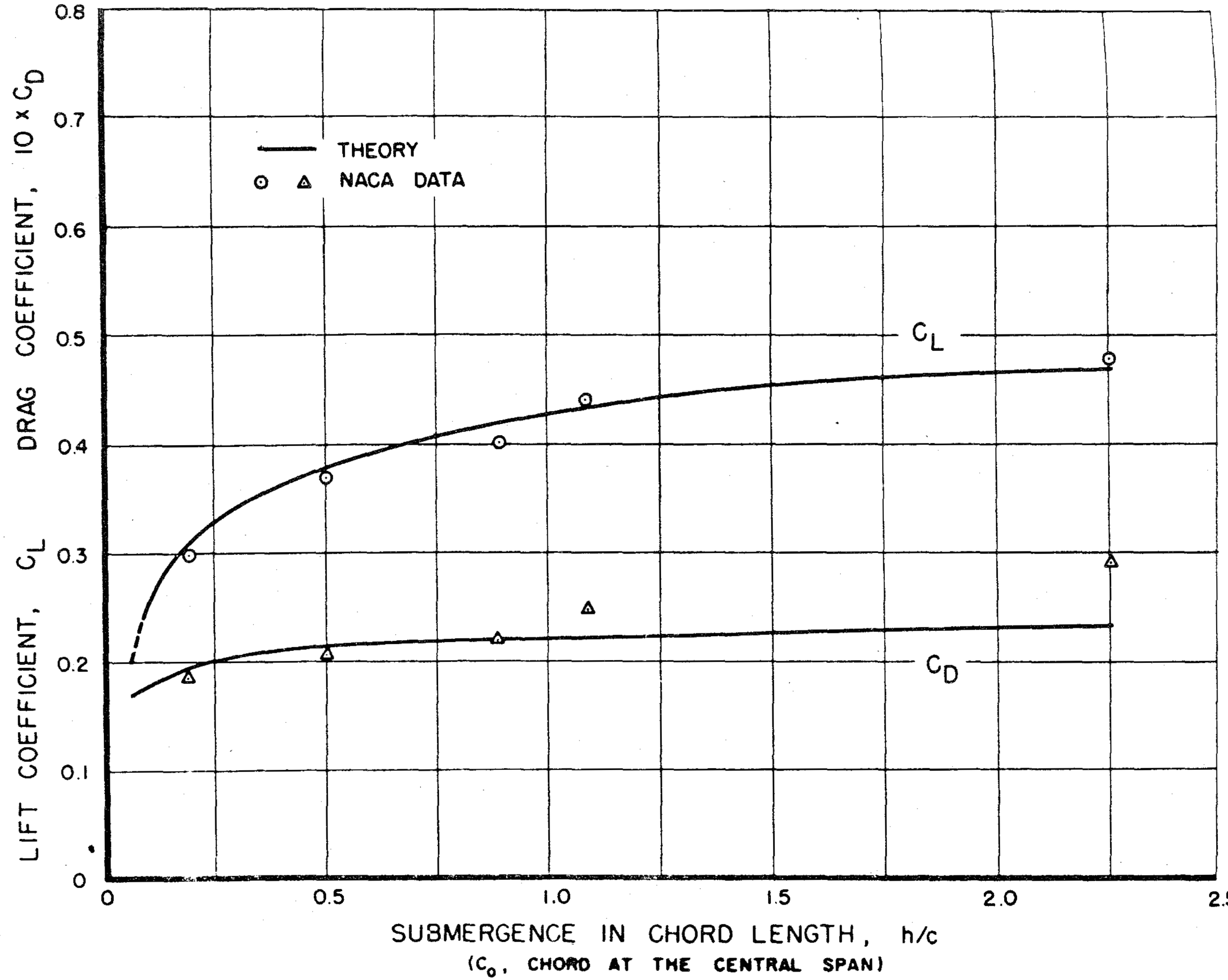

Fig. 13 - The effect of depth of submergence on C_L and C_D (three-dimensional, noncavitated flow).

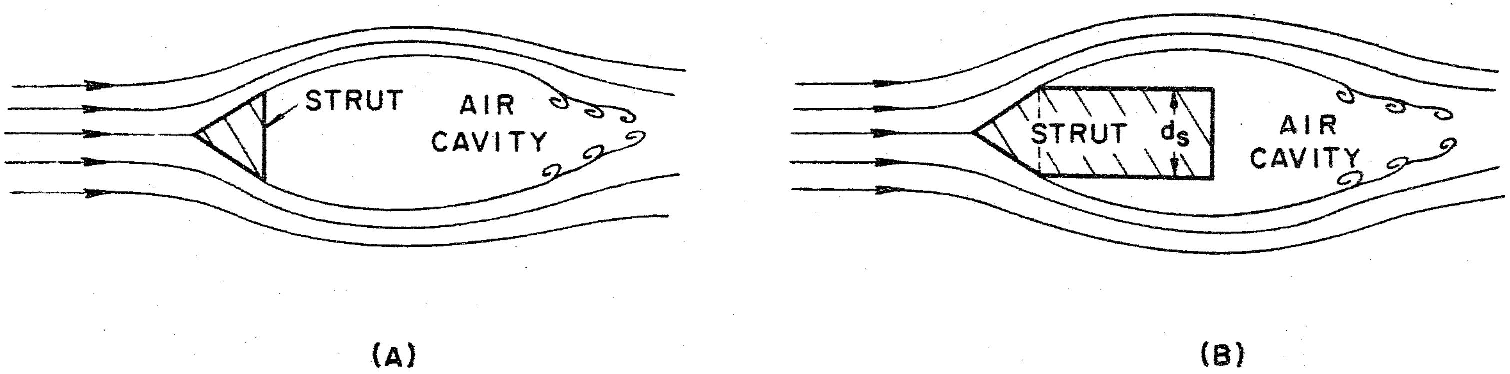

Fig. 14 - Struts with air cavity.

range of depths less than two chords, C_D decreases gradually and C_L decreases comparatively rapidly with decrease in depth. Moreover, the agreement with the experiments evidently indicates that the lifting line theory gives an accurate estimate of the effects of finite span and free surface for hydrofoils of large aspect ratio when operated without cavitation.

Now for a fully cavitating hydrofoil of large but finite span submerged at finite depth, it is conjectured that the lifting line theory can be applied if one assumes that the cavitating hydrofoil can be replaced by a bound and trailing vortex system. The lift distribution along the lifting line is the same as the lift distribution, after integration with respect to the chord of the hydrofoil, along the span direction. The induced velocity field will result from three causes: the vortex system representing the lifting line, the free surface pressure condition and the wave formation on the water surface. The local flow, with the induced velocity so determined, around the cavitating hydrofoil at each section perpendicular to the span can be considered as that of a two-dimensional flow around a cavitated hydrofoil of infinite span without free water surface. The data of such flow are given by the above two-dimensional theory or can be obtained by experiment for some general geometric shapes, provided that the Reynolds number and cavitation number are the same. In any event, we would expect to obtain a result for the fully cavitating hydrofoils quite similar to that described above for the noncavitating case. That is, C_L and C_D change smoothly from their three-dimensional cavitating values (without water surface) to the planing values within a submergence depth of about two (to four) chord lengths. The effect of finite span alone could also be calculated for the fully cavitating hydrofoil under these assumptions. It should be remembered that the replacement of the cavitating hydrofoil by a lifting line in this particular case is only an assumption which must be verified by experiments. But if it is correct, it then indicates that the two-dimensional comparison of cavitating and noncavitating hydrofoils should give reasonable estimates of the relative merits of the corresponding three-dimensional configurations. Further work on this point certainly seems necessary.

V. CAVITATING AND NONCAVITATING STRUTS AND APPENDAGES

The next problem for consideration is the nonlifting, surface-penetrating strut, such as would be used to support the hull on its underwater lifting hydrofoils. As in the preceding case, a simplification will first be made by assuming that the strut flow is two-dimensional and that the effect of the free surface which it penetrates is negligible. Afterwards the data are corrected for these effects. This approximation is reasonably accurate if the ratio of submergence depth to chord is fairly large. It seems likely that most practical applications of strut will lie in this range so that the following comparisons which are simplified in the above manner may be directly applicable in some instances.

It is known that when a vertical bluff strut moves horizontally at high speeds with its top end projecting through the free water surface, the cavity region just aft of the strut will in general be ventilated by air drawn down from the surface.[13] If a horizontal section is taken through the strut the flow might be assumed to be two-dimensional if the section taken is not too near the water surface or the lower end of the strut. (For example, see Fig. 14a where the strut has a wedge shape). Under these conditions the drag can be calculated simply from two-dimensional cavity theory.

In an air-ventilated flow the effective cavitation number for the flow section at a depth y below the free surface is easily shown to be

$$\sigma(y) = \frac{g\,y}{U^2/2} , \tag{19}$$

where g is the acceleration due to gravity. The average value of $\sigma(y)$ for a vertical strut at total submergence depth h is then

$$\bar{\sigma} = \frac{1}{h}\int_o^h \sigma(y)\,dy = \frac{gh}{U^2} . \tag{20}$$

Thus for small submergences and high speeds the effective values of $\bar{\sigma}$ are very low. For a strut symmetrical with respect to a central vertical plane the drag coefficient would then be[13]

$$C_D = (1+\bar{\sigma})\, C_{D_o} = C_{D_o}\,(1 + gh/U^2) \tag{21}$$

where C_{D_o} is the drag coefficient corresponding to $\bar{\sigma} = 0$. For a wedge-shaped strut the value of C_{D_o} is shown as a function of wedge angle in Fig. 15 (which is reproduced here from Ref. 13, Fig. 10).

On the other hand, when the cavity is kept from being ventilated, the corresponding value of σ is always greater than gh/U^2 (the σ for the ventilated case, see Eq. 20), since the vapor pressure is less than atmospheric pressure. Consequently the drag in the ventilated case (Eq. 21) is the lowest possible value of the drag for the strut. Sometimes this ventilating phenomenon is described by saying that the base pressure is increased, resulting in a reduction in "base drag". In some cases the reduction of the base drag may be a very large effect, amounting to a reduction of an appreciable fraction of the unventilated value (see Ref. 13, Fig. 11). The magnitude of improvement can be estimated in any instance where there is data for the bluff shape in full cavity flow. For example, if the strut has a simple wedge shape, Fig. 15 can be used for this purpose.

However, if air ventilation is allowed to extend downward into the cavity on the hydrofoil which is attached to the lower end of the strut, then the lift of the hydrofoil decreases due to decrease in σ, as discussed in Sec. III-b. Thus, an efficient operation at high speeds such as to obtain low strut drag yet without sacrificing the hydrofoil lift would necessitate some sort of horizontal partitions separating these two cavity regions. The drag of the strut portion in the unventilated vapor cavity can also be computed with the aid of cavity theory, but the vapor cavitation number σ must be used in Eq. 21. For struts of such configuration, it is possible that the vapor cavity dimensions will increase, as the speed is increased, until they exceed the size of the partition. Air will then ventilate into the vapor cavity, producing a sudden change in hydrodynamical forces and possibly forming air cavities in other regions where only vapor cavities previously existed. In many instances these

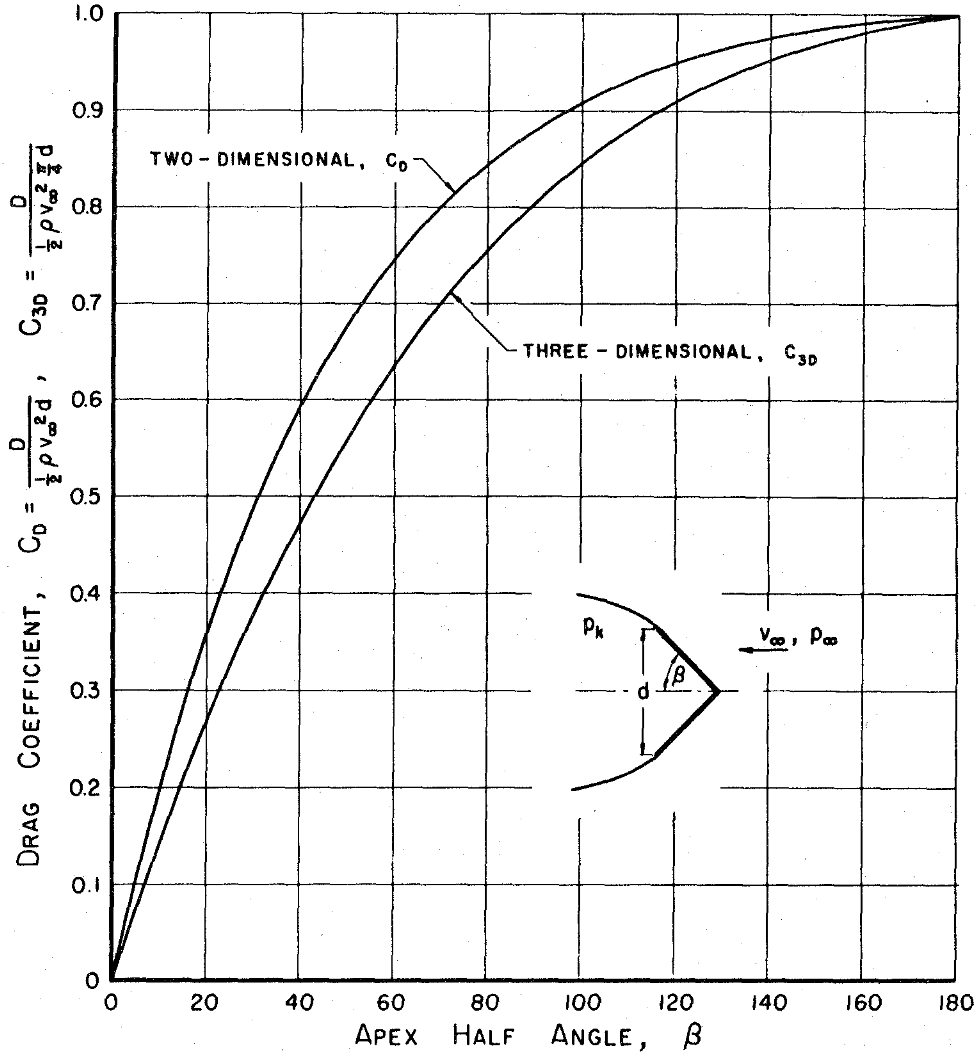

Fig. 15 - The two- and three-dimensional drag coefficients at $\sigma = 0$ as a function of β.

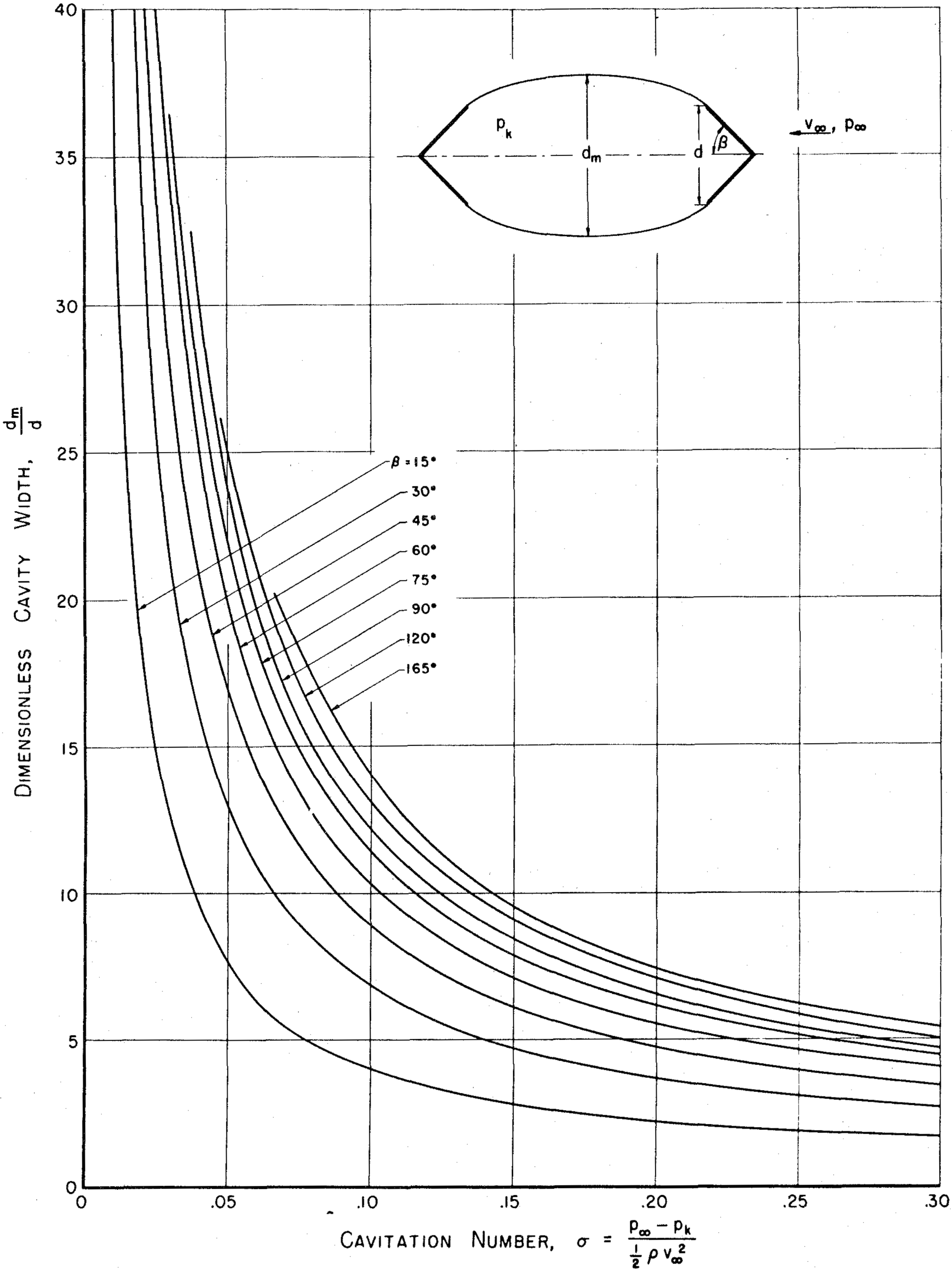

Fig. 16 - The width of a two-dimensional cavity.

effects might produce undesirable or even dangerous consequences such as sudden reduction in the hydrofoil lift. Naturally the designer should have such effects in mind in deciding whether they should be avoided or not in any particular case.

After the cavity flow is established, ventilated or not, it is possible to extend the strut into part of the cavity region, as shown in Fig. 14-b, without changing the force on the strut. Since this extension of the strut allows more structural strength without any penalty in drag, the dimensions of the cavity would be of interest to the designer since he can then design the strut profile to take advantage of this effect. The general features of the cavity shape are fairly well understood, and in the case of wedges, the principal dimensions have been computed theoretically.[14] Some of these results are reproduced here as Figs. 16-18. From these data an interesting fact, which may be useful in preliminary design considerations, can be noted that <u>for a given nose drag</u> the general cavity size depends primarily on the cavitation number. This relationship, which is also true for three-dimensional cavities, can be seen from Fig. 18, where the over-all cavity length-width ratio for wedges of different wedge angles is shown to depend only on σ to a good approximation. However, the local cavity shape near the nose depends to some extent on the specific nose geometry. For example, either an acute wedge nose or a flat plate nose can be used to obtain a certain size of cavity, although the plate nose must have a smaller base width. To be more specific, at $\sigma = 0.1$ the width of a flat plate must be approximately one-third of the base width of a 15° half-vertex angle wedge in order that these two cavities are of the same over-all width and length. This statement can be easily verified from Fig. 16-18. In view of this relationship the designer has considerable leeway in choosing the nose shape to throw a given cavity, although the nose size must be altered consistently. Since the strut portion which is interior to the cavity does not affect the hydrodynamics there is also considerable latitude of choice in its profile. However, it is certainly desirable to shape the strut in a manner that will favor low drag when the strut is running fully wetted at very low speeds.

The viscous drag coefficient of a fully wetted strut can be estimated by using Eq. (8) for the corresponding range of Reynolds number.

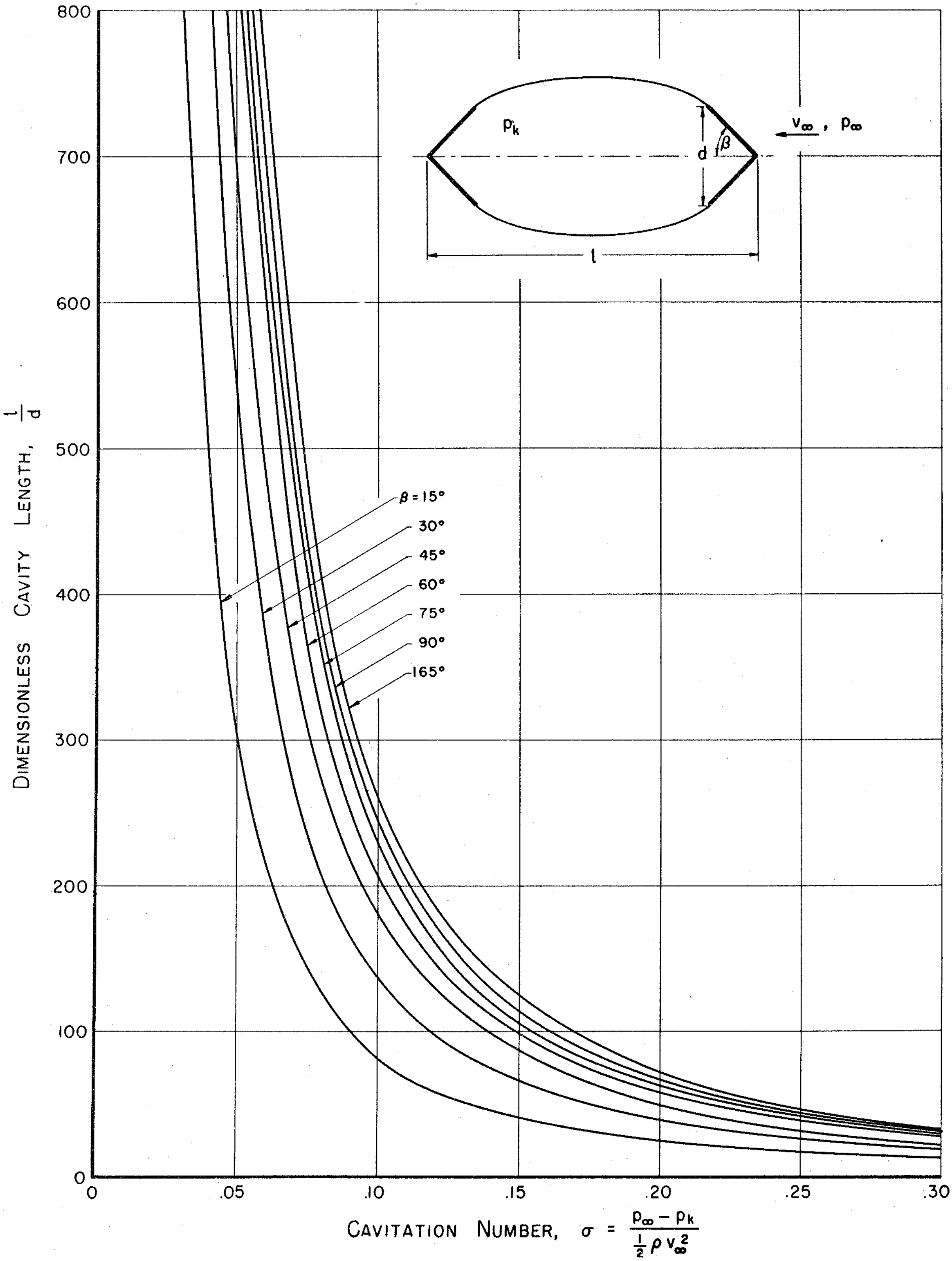

Fig. 17 - The length of a two-dimensional cavity.

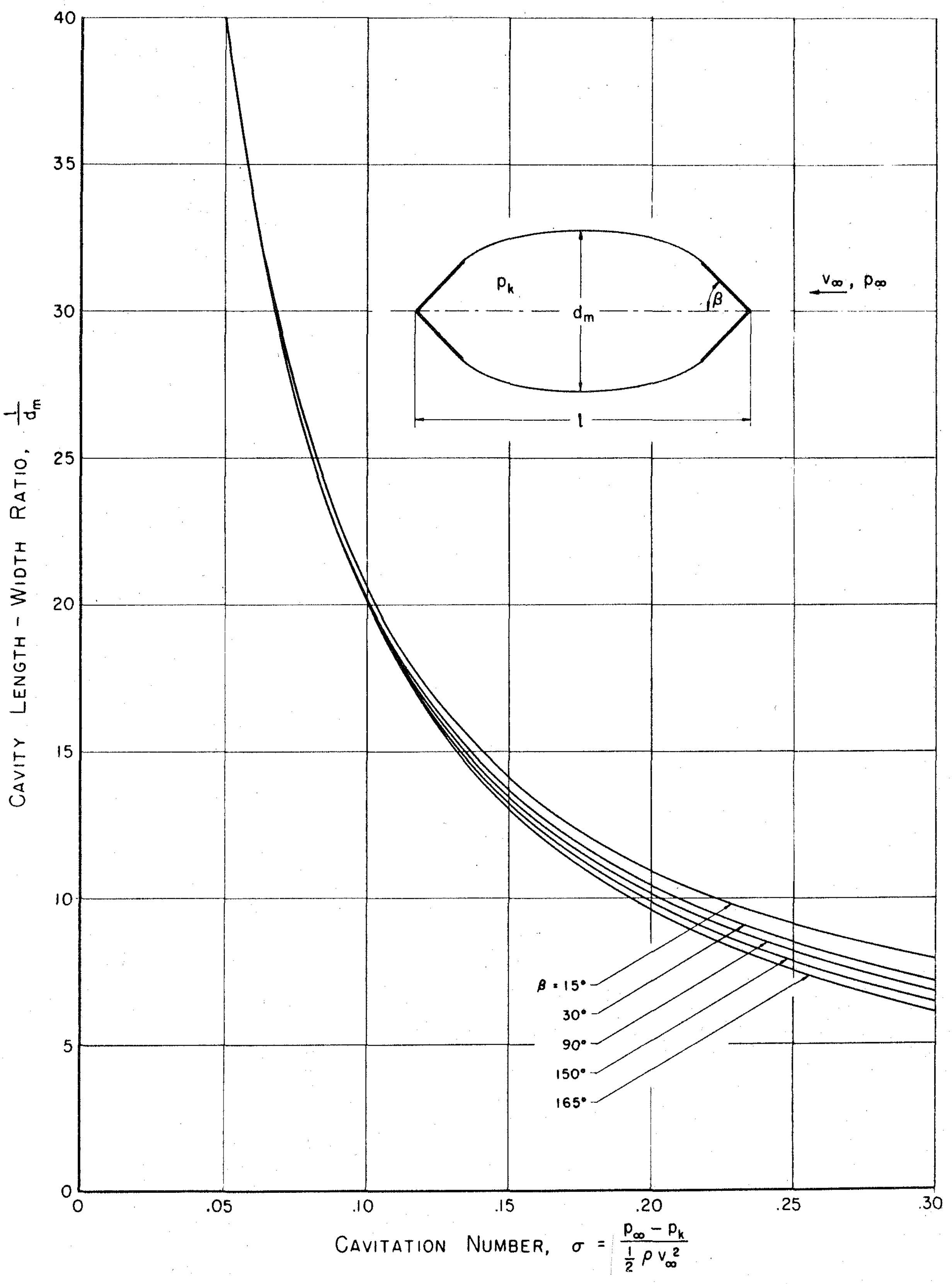

Fig. 18 - The length-width ratio of a two-dimensional cavity. At very low σ, the ratio is independent of β, being asymptotic to $2/\sigma$.

It will be further assumed here that the skin friction drag of a two-dimensional streamline-shaped strut has the same value as that on a flat plate of equal area, although experimentally the former is found slightly greater than the latter at the same Reynolds number. Thus, the total skin friction drag experienced by a strut of wetted surface S is approximately

$$D_f = \frac{1}{2} \rho U^2 C_f S \tag{22}$$

where C_f is given by Eq. (8) or some equivalent formula.

In this connection we mention another interesting fact concerning the fully cavitated and fully wetted operations of streamline-shaped struts. For simplicity we consider here only the struts with a flat plate nose of width d. Suppose that the strut is shaped in such a way that it almost fills the cavity at a certain value of cavitation number, say σ_i. The value of σ_i is called the incipient cavitation number of this strut, since such a strut would not, in principle, cavitate at any value of σ higher than σ_i. For $\sigma < \sigma_i$, the cavity is fully developed and the strut then experiences a cavity drag, which is the total drag in this case, equal to

$$D_\sigma = \frac{1}{2} \rho U^2 C_D(\sigma)\, d \tag{23}$$

where C_D is the well-known result given by Eq. (14),

$$C_D(\sigma) = 0.88(1 + \sigma) \quad \text{for} \quad \sigma < \sigma_i . \tag{24}$$

Now $C_D(\sigma)$ is at least 0.88 while C_f, given by Eq. (8), is of the order of 0.005 for the prescribed range of Re; the resulting value of $C_D(\sigma)/C_f$ is thus of the order of 150. Since D_f is based on the wetted length S whereas D_σ is referred to the base width d, it is clear that for (S/d) less than the above order of magnitude for $C_D(\sigma)/C_f$, the drag will increase as the cavity starts to establish. However, the opposite may also occur (that is, the drag will decrease when cavitating) for struts having larger values of (S/d). At one point, the drag would be equal in either case. To find this criterion, we refer to Ref. 14 that at $\sigma = \sigma_i$ the cavity is approximately of the form of an ellipse, with a major axis ℓ_i and minor axis d_{mi} where for σ_i small

$$\frac{\ell_i}{d} \cong \frac{16}{4+\pi}\,\sigma_i^{-2} \;, \qquad \frac{d_{mi}}{d} \cong \frac{8}{4+\pi}\,\sigma_i^{-1} \;.$$

Hence the circumference of the ellipse is

$$S \cong 2\ell_i \cong 4.49\,\sigma_i^{-2}\,d \;.$$

Therefore we obtain

$$\frac{D_f}{D_\sigma} \cong \frac{C_f}{C_D(\sigma)}\;\frac{4.49}{\sigma_i^{\,2}} \;.$$

The value of σ_i which gives the above ratio unity can readily be determined for given C_f. In general, this critical value of σ_i is of the order of 0.14, corresponding to a fairness ratio $\ell_i/d_{mi} \cong 15$. In practical applications, however, the fairness ratio of struts is in general much less than this value in order to favor low drag when the strut is running fully wetted at low speeds.

When the appendages contain some slender or axisymmetric bodies, the characteristics of such bodies at cavity-running or non-cavity running conditions are quite similar to those of struts as discussed above. No further elaboration will be made here on these points.

VI. REMARKS ON PRACTICAL CONSIDERATIONS

While the preceding sections have outlined the performance characteristics of the individual components, the design of a practical hydrofoil boat system still requires consideration of combinations of struts and hydrofoils, and possibly other submerged components such as the propulsive system. It seems feasible to proceed by estimating first the performance of the various components separately and then to make an allowance for interference or interaction effects so that the over-all performance can be estimated. For completely wetted foils some of these interactions might be estimated from available aerodynamic data. For example, the intersection of the hydrofoil with the supporting strut might be designed by this approach, much the same as the wing-fuselage interference in aerodynamics.

For the fully cavitating hydrofoil, the strut-hydrofoil intersection seems easier to design since all the structure and fairings may be internal to the cavity and hence will not cause any hydrodynamic interference. On the other hand, if during the development of cavity flow the strut or the hydrofoil nearly fills the cavity, peculiar effects may be observed because of the splash from various spray sheets striking the surfaces of the hydrofoil or strut in an unexpected fashion. In the present state of the art it would be advisable to check the design of all intersections in model experiments where full view can be obtained of the flow in the vicinity of the intersections. Relatively little is known as to the magnitude of the interactions for fully cavitating strut-hydrofoil systems, and more information on this problem would be of value.

Another source of interference effects is the intersection of the support struts and the free surface. The free surface effect on non-cavitating struts is essentially to generate a gravity wave on the free surface. The wave resistance of such struts can be calculated by applying the classical Michell's thin ship theory.[15] Except for this effect, the free surface interference seems otherwise not appreciable. Some recent experiments on cavitating struts also have indicated that the intersection does not introduce any radical changes from the two-dimensional estimates.[13] It may be mentioned here that if the lower end of a strut passes into a cavity, the end flow situation is almost identical with that at the free surface, and the interference effect should be of the same nature.

VII. REMARKS ON ADVANTAGES AND DISADVANTAGES OF CAVITY RUNNING; APPROXIMATE ECONOMY COMPARISON

The trend toward higher speed operations and the practical limitation on submergence of the hydrofoil system indicate that cavitation will be more and more difficult to avoid in future designs. If it must be avoided, the only course which the designer can follow is to shape the profiles of the various submerged hydrofoils and struts so that the local pressure is never less than the vapor pressure at the maximum speed of the boat. In practice this means that the higher the design speed, the more slender must be the hydrofoils and struts. Unfortunately, as pointed out in a preceding section, the increasing slenderness of the foils and struts is rather disadvantageous from the standpoint of structural design, and beyond a certain point there is a consequent penalty in drag. It may be that in this case it is more profitable to utilize a cavity running design.

Even though the designer has constructed all the parts so that vapor cavitation will be avoided within a prescribed speed range and limited attack angles of the hydrofoils, there is still, in some cases, the possibility that air ventilation could occur. For example, if the boat is to operate in rough water, additional consideration must be given to the possibility of an accidental exposure of a hydrofoil to the atmosphere and consequent cavity formation by air ventilation. Such a cavity might persist even after the hydrofoil was returned to its normal running depth if the air cavity could communicate in some fashion with the atmosphere. Similarly, the ventilation of air into an area where a vapor cavity had formed might result in a vastly enlarged cavity which could then ventilate to the atmosphere and maintain an air cavity after the boat had slowed down to a speed at which vapor cavitation ordinarily could not initiate. Possibilities of this kind indicate that in the execution of the design, for noncavity running only, the cavity-running effects should still be considered because of the likelihood that such air cavities might occur accidentally.

It may be pointed out here also that it is not really necessary to run both supporting struts and lifting hydrofoils in the same flow regime.

For example, one might operate the lifting hydrofoils with full vapor cavitation while having no cavitation on the struts. Or, one might consider running the hydrofoils without cavitation but allowing air ventilation on the upper part of the struts. The consideration of all the possibilities actually would amount to a system analysis which might uncover a more effective approach than one would otherwise consider.

Next, a comparison from the economy point of view, much the same as the operation research applied to aircraft in commercial service, will be calculated in a simplified manner, thus providing a crude criterion for cavitated operation. A complete analysis of this kind would require consideration of all relevant factors, such as capital investment, labor costs, administration and maintenance expenses, together with factors of a purely hydrodynamic nature. To be more precise, the hydrodynamic part of the problem is referred to as the determination of the lift and drag of the hydrofoil (or a system of hydrofoils) operating in these two flow regimes, with the effects of free water surface, finite span, interaction between different parts, and so forth, all taken into account. The lift estimates the gross weight which the hydrofoil boat is capable of carrying and the drag determines the power required to provide a given lift, or the energy consumption in traveling a given distance with a prescribed load. A complete survey considering all of the relevant factors would certainly be useful in a specific design study. Such calculations, if undertaken early in design, may likely bring out the need for more specific data. However, while this paper emphasizes only the hydrodynamic aspects of the entire problem, our present interest is to single out the important factor of the energy consumption in order to obtain some concrete notion of the general trend for this one parameter.

It should first be remarked that high efficiency is not always a predominant factor of concern in high speed operation. For instance, it might be profitable to operate a boat at high speed using fully cavitating hydrofoils, even if there were some losses in efficiency, because a given number of runs could be made in less time or by fewer craft. Suppose that, with hydrofoils cavitated, an available engine can propel a hydrofoil craft of given gross weight three times as fast as for a similar noncavitating craft. If the problem of comparing the respective capital investment, labor costs, and so forth, is disregarded for the moment, the question is then whether the total energy consumed in cavitating operation is more, or less, than three times that required for noncavitating operation to travel the same distance.

For simplicity, let us confine ourselves to the case of a flat plate hydrofoil, for which two-dimensional hydrodynamic information is presently available. In this simplified calculation, the analysis will be based on two-dimensional theory with some account taken for the effects of skin friction and wave drag. Because in this general study the detailed configuration of the hydrofoil system is not definite, the effects of finite span and possible interaction phenomena are not considered. In any specific case, these effects could be estimated separately.

For a given angle of attack there is a critical speed, U_c, at which the cavity starts to develop. Suppose that a flat plate hydrofoil of plan area A_o, moves fully wetted at speed U_o, less than U_c ; and another hydrofoil of plan area A_1, runs fully cavitated at U_1 greater than U_c. The total lift they experience is then, respectively,

$$L_o = \frac{1}{2}\rho U_o^2 A_o C_{L_o} , \qquad L_1 = \frac{1}{2}\rho U_1^2 A_1 C_{L_1} .$$

Now an appropriate basis for comparison would be that the cavitating and noncavitating hydrofoils both exert the same lift, so as to support an equal load. When $L_o = L_1$, we then have

$$\frac{U_o^2 A_o}{U_1^2 A_1} = \frac{C_{L_1}}{C_{L_o}} . \tag{25}$$

It also follows from this result that the ratio of the drag in these two cases is

$$\frac{D_o}{D_1} = \left(\frac{C_{L_1}}{C_{D_1}}\right) \Big/ \left(\frac{C_{L_o}}{C_{D_o}}\right) \tag{26}$$

which is also the ratio of energy required to travel the same distance for these two different operations, that is

$$E_o/E_1 = D_o/D_1 . \tag{27}$$

The ratio of the power required to maintain these operations is then

$$\frac{P_o}{P_1} = \frac{D_o U_o}{D_1 U_1} = \left(\frac{U_o}{U_1}\right)\left(\frac{C_{L_1}}{C_{D_1}}\right) \Big/ \left(\frac{C_{L_o}}{C_{D_o}}\right) \quad . \tag{28}$$

Moreover, the quantity U_o/U_1 can be expressed in terms of the "cavitation number" as follows:

$$\sigma_o = (p_\infty - p_\sigma)/(\tfrac{1}{2}\rho U_o^2) \,, \qquad \sigma = (p_\infty - p_\sigma)/(\tfrac{1}{2}\rho U_1^2)$$

so that

$$\frac{U_o}{U_1} = \left(\frac{\sigma}{\sigma_o}\right)^{1/2} \tag{29}$$

Here σ is the cavitation number for the cavitating flow while σ_o is merely the corresponding value of σ for the actually noncavitating flow. In general the critical speed U_c corresponds to a cavitation number about unity. In order to assure the noncavitating operation, σ_o in general should be greater than 1.5. For simplicity we shall approximate the quantities in the above equations by using the two-dimensional values of C_L and C_D of the flat plate. The important features of the result so obtained should remain even when other effects such as the influence of free surface and finite span, etc., are considered. In two-dimensional problems, the available data on C_L and C_D enable us to compare (U_o/U_1), (D_o/D_1) and (P_o/P_1) for different σ's. One example will be worked out here by assuming that both the cavitating and noncavitating plate are held at $\alpha = 6^o$. This restriction is imposed here merely for convenience and is really not necessary. It would be equally adequate to compare these two hydrofoils when they are held at unequal values of α. Now with some interpolation in Fig. 3, it is indicated that the plate will be fully cavitated for $\sigma = 0.5$. In order to bring the comparison on a more realistic basis, an estimated wave drag and skin friction drag will be added to the cavity drag. It is then reasonable to assume the following value

$$C_{D_o} = 0.02 \,, \qquad C_{D_1} = 0.01 + C_{D_c} \tag{30}$$

where C_{D_c} is the drag coefficient resulting solely from the cavity flow.

By using the value of C_{L_1} and C_{D_c} given in Figs. 3 and 5, Eqs. (26) - (29) are then plotted against σ in Fig. 19 for two values of σ_o, namely $\sigma_o = 2$ and $\sigma_o = 4$ which correspond approximately to $U_o = 0.707\ U_c$ and $0.5\ U_c$.

From this figure it becomes clear that under the present comparison basis cavitating operation is of advantage for $\sigma < 0.19$ when $\sigma_o = 4$ and for $\sigma < 0.07$ when $\sigma_o = 2$, because at $\sigma_o = 4$, E_1/E_o is less than U_1/U_o for $\sigma < 0.19$. In other words, when the cavitating hydrofoil travels with U_1 five times as fast as the noncavitating one, the total energy required is less than five times the energy spent in noncavitating operation. The result also indicates that the smaller the value of σ, the more advantage there is for cavitating operation. However, higher powered engines are required for motions at smaller values of σ, as can be seen from Fig. 19.

Thus it is up to the designer to decide whether the cruising speed should be in the noncavitating or cavitating regime. If the noncavitating operation is chosen, then some attention should be given to designing a hydrofoil of such geometric shape that it gives the highest possible critical velocity U_c and an almost constant pressure distribution on the suction side of the hydrofoil.[16] On the other hand, if cavitating hydrofoils are to be used, then it may be more economical to operate at higher speeds.

It is also worthwhile to consider the economic problem from another viewpoint. While it is a standard aeronautical practice in comparing airfoils to use the lift-drag ratio as a measure of the relative merits of different profiles, it is natural to apply the same criterion in the hydrofoil problem. If cavitation is not a problem this is a legitimate approach, but if cavitation must be considered then it is more reasonable to compare lift-drag ratios at the same cavitation number. From this point of view the "best" hydrofoil profile would be that which has the highest lift-drag ratio at a given cavitation number.

VIII. SUMMARY AND CONCLUDING REMARKS

The salient points of the previous discussions can be summarized as follows:

1. The use of hydrofoils with large camber to increase the lift coefficient is more effective in cavitating operation than in fully wetted conditions, especially for small angles of attack. The increase in C_D due to an increment in camber is rather insignificant relative to the gain in C_L.

2. When the cavitation number σ alone is varied, both C_L and C_D increase with increasing σ. Their rates of increase are much more marked when the angle of attack α is small. With the free stream velocity U assigned, σ can be made larger by submerging the hydrofoil at greater depths.

3. The lift coefficient of fully cavitating hydrofoils is rather insensitive to variations of angle of attack when compared with fully wetted hydrofoils. For a hydrofoil cavitating at a given value of σ, it is advisable to keep α slightly greater than its critical value α_c so that there is little danger of large fluctuations in the lift caused by fluctuations of approaching flow direction.

4. When a hydrofoil is operated at a speed very close to the critical cavitating value, the fluctuation of lift and drag due to small variations of the relative velocity could be of harmful magnitude. Therefore, this range of operation should be avoided or be passed through as rapidly as possible.

5. If the design is aimed at noncavitating operation, the emphasis is then to design a hydrofoil which would give the highest possible critical cavitating speed. If the cavitating hydrofoil is to be used, then advantage may as well be taken of operation at higher speeds.

6. The motion of a cavitating hydrofoil at shallow submergence is much smoother than that of a noncavitating hydrofoil because the loss of lift when a cavitating hydrofoil becomes a planing surface is much smaller than that of a noncavitating hydrofoil.

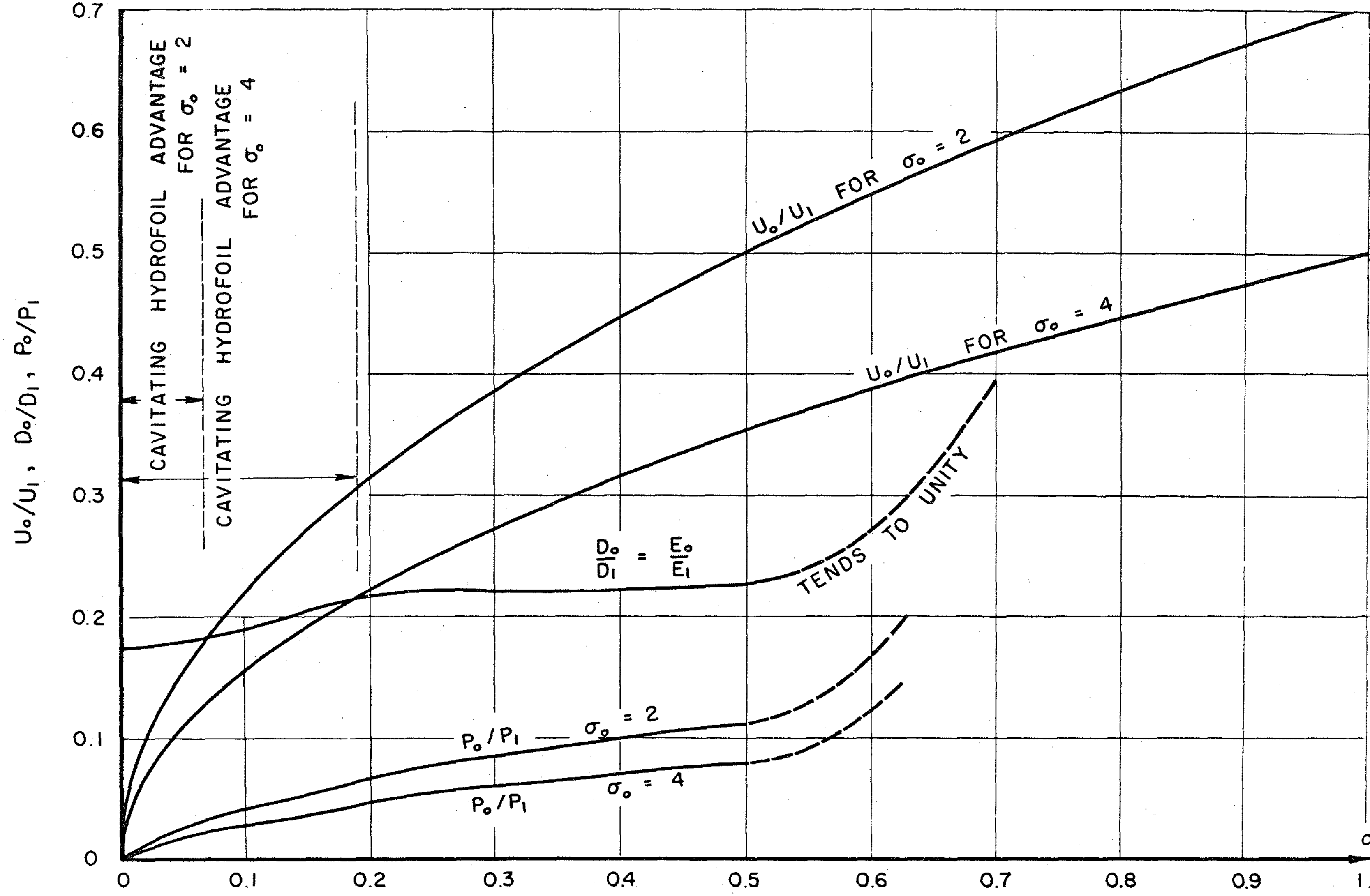

Fig. 19 - Economy comparison of cavitating and noncavitating operations.

The interest in the hydrofoil boat for practical applications in naval architecture is primarily due to its capabilities for low drag at comparatively high speed. But at high speeds the hydrofoil system is inherently susceptible to cavitation. Thus far engineering experience with cavitation phenomena has been concerned mainly with the cases where the presence of cavitation leads to very undesirable effects, such as loss of efficiency and mechanical damage. For this reason there has been a natural tendency to regard cavitation as a deleterious feature which should be avoided at all costs. So far as high-speed hydrofoils are concerned, however, the preceding discussion indicates that in some cases cavitation might be an alternative which is by no means without advantages.

On the other side of the ledger, it should be noted that the cavity-running design has a much smaller backlog of experience on which the designer may rely. Thus, the fully wetted foil can be treated in the first approximation by standard airfoil methods, and the vast store of experiments on airfoils can be used to predict hydrofoil behavior with proper considerations of the effect of free surface and finite span. It must be remembered, however, that the requirement of avoiding cavitation at higher speeds presents a very difficult design condition, one which available airfoil design data do not adequately solve. Hence there is no justification for assuming that the design problem for a high-speed boat can be solved by a straightforward application of existing data. Rather, both the cavitating and noncavitating designs really need more data than are now available for their proper execution. It is hoped therefore that the present report will be of assistance in outlining some of the important problems for further work.

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